Trade-off between Science Data Requirements and Lidar Engineering Designs

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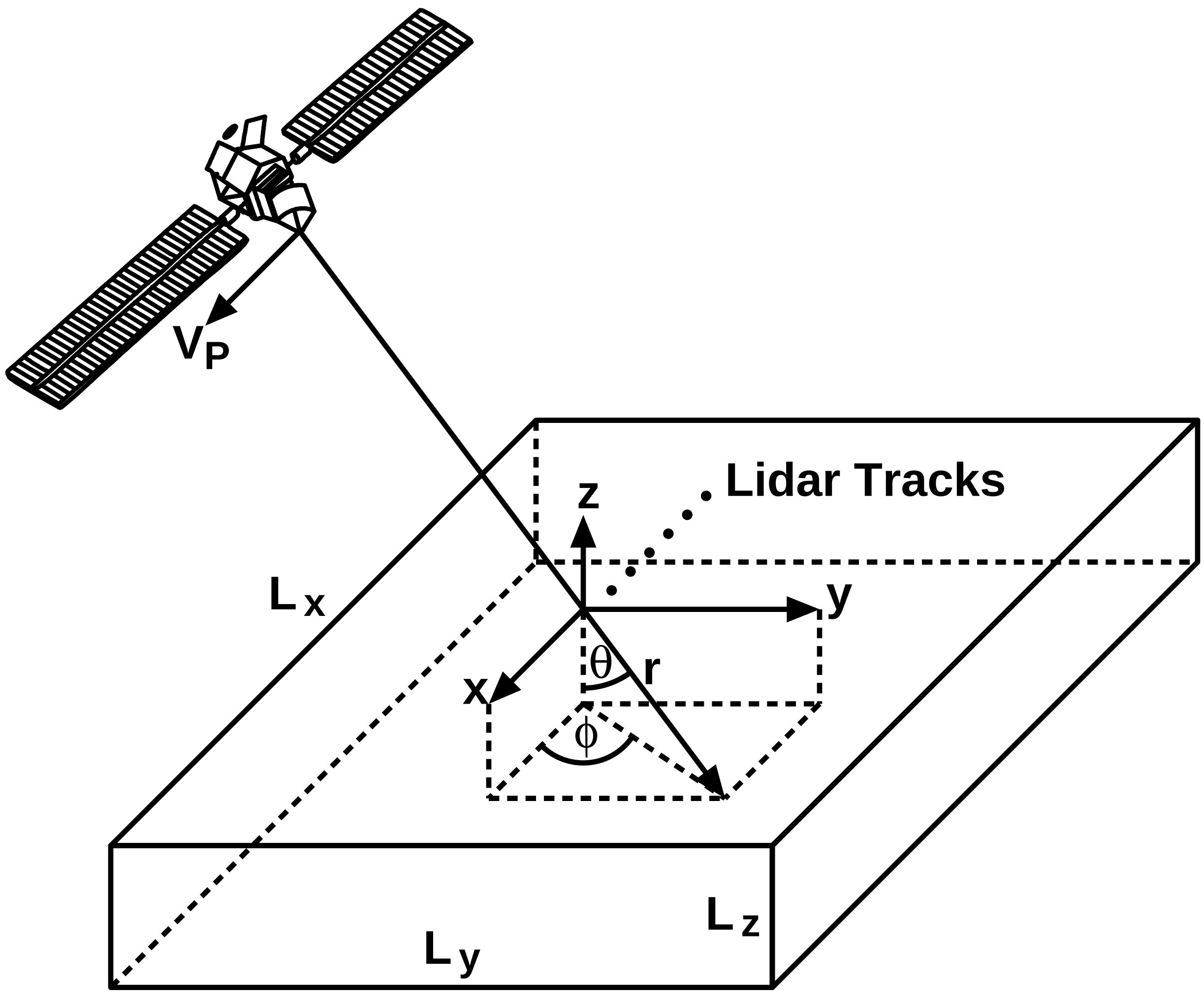
Schematic of a Space-Based Doppler Lidar



DEFINITION OF VELOCITY ACCURACY

- Velocity field x is random

- Definition of velocity accuracy
  requires definition of truth "x_{truth}"

- Accuracy composed of bias and random error e

  $\hat{x} = x_{truth} + e + BIAS$

- Bias of estimate = $< \hat{x} - x_{truth} >$
  $< x >$ denotes ensemble average of x over
  the random wind field

- Total Random Error $\Sigma^2 = < [\hat{x} - < \hat{x} >]^2 >$

- $\Sigma^2 = \delta^2 + \sigma^2$

δ - representative error

σ - instrument and estimation error

HORIZONTAL VELOCITY MEASUREMENTS FROM TWO FORWARD AND AFT LOS MEASUREMENTS

- truth (u_{truth}, v_{truth}) is defined as the average of the horizontal velocity vector (u,v) over the Target Sample Volume (TSV)

- estimates $(\hat{u}, \hat{v})$ produced from two LOS estimates $\hat{v}_{for}$ and $\hat{v}_{aft}$ assuming zero vertical velocity

$$\hat{u} = \frac{\sin\phi_{aft}\sin\theta_{aft}\hat{v}_{for} - \sin\phi_{for}\sin\theta_{for}\hat{v}_{aft}}{\cos\phi_{for}\sin\theta_{for}\sin\phi_{aft}\sin\theta_{aft} - \sin\phi_{for}\sin\theta_{for}\cos\phi_{aft}\sin\theta_{aft}}$$

$$\hat{v} = \frac{\cos\phi_{aft}\sin\theta_{aft}\hat{v}_{for} + \cos\phi_{for}\sin\theta_{for}\hat{v}_{aft}}{\cos\phi_{for}\sin\theta_{for}\sin\phi_{aft}\sin\theta_{aft} - \sin\phi_{for}\sin\theta_{for}\cos\phi_{aft}\sin\theta_{aft}}$$

- Representation of estimates

$$\hat{u} = u_S + e_u + BIAS_u \quad , \quad \hat{v} = v_S + e_v + BIAS_v$$

u_S, v_S - spatial average over measurement plane

e_u, e_v - random errors from LOS random errors

$BIAS_u, BIAS_v$ - Bias from LOS bias

TOTAL RANDOM ERROR

ALONG-TRACK COMPONENT

$$\Sigma_u^2 = \delta_u^2 + \sigma_u^2$$

$\delta_u^2 = < [u_S - u_{truth}]^2 >$ - sampling error

$\sigma_u^2 = < e_u^2 >$ - random error from LOS error

ACROSS-TRACK COMPONENT

$$\Sigma_v^2 = \delta_v^2 + \sigma_v^2$$

$\delta_v^2 = < [v_S - v_{truth}]^2 >$ - sampling error

$\sigma_v^2 = < e_v^2 >$ - random error from LOS error

- Critical tradeoff between sampling error and random error

HORIZONTAL VECTOR MEASUREMENT FROM TWO OVERLAID FOR AND AFT TRACKS

- $\hat{u} = (\hat{v}_{for} - \hat{v}_{aft})/2\cos(\phi)\sin(\theta)$
- $\hat{v} = (\hat{v}_{for} + \hat{v}_{aft})/2\sin(\phi)\sin(\theta)$
  - θ = nadir angle
  - ϕ = azimuth angle

- sampling error

$$\delta_u^2 = < [\bar{u} - u_{truth}]^2 >$$

$$\delta_v^2 = < [\bar{v} - \bar{w}\ \mathrm{cotan}\theta/\sin\phi - v_{truth}]^2 >$$

$\bar{u}$ - spatial average of u over measurement plane
$\bar{v}$ - spatial average of v over measurement plane
$\bar{w}$ - spatial average of w over measurement plane

- sampling error requires scan geometry and a spatial description of the wind field

- troposphere has Kolmogorov spatial spectrum

RANDOM ERROR FROM LOS RANDOM ERROR

- **LOS random instrument errors are uncorrelated and equal (σ_{LOS})**

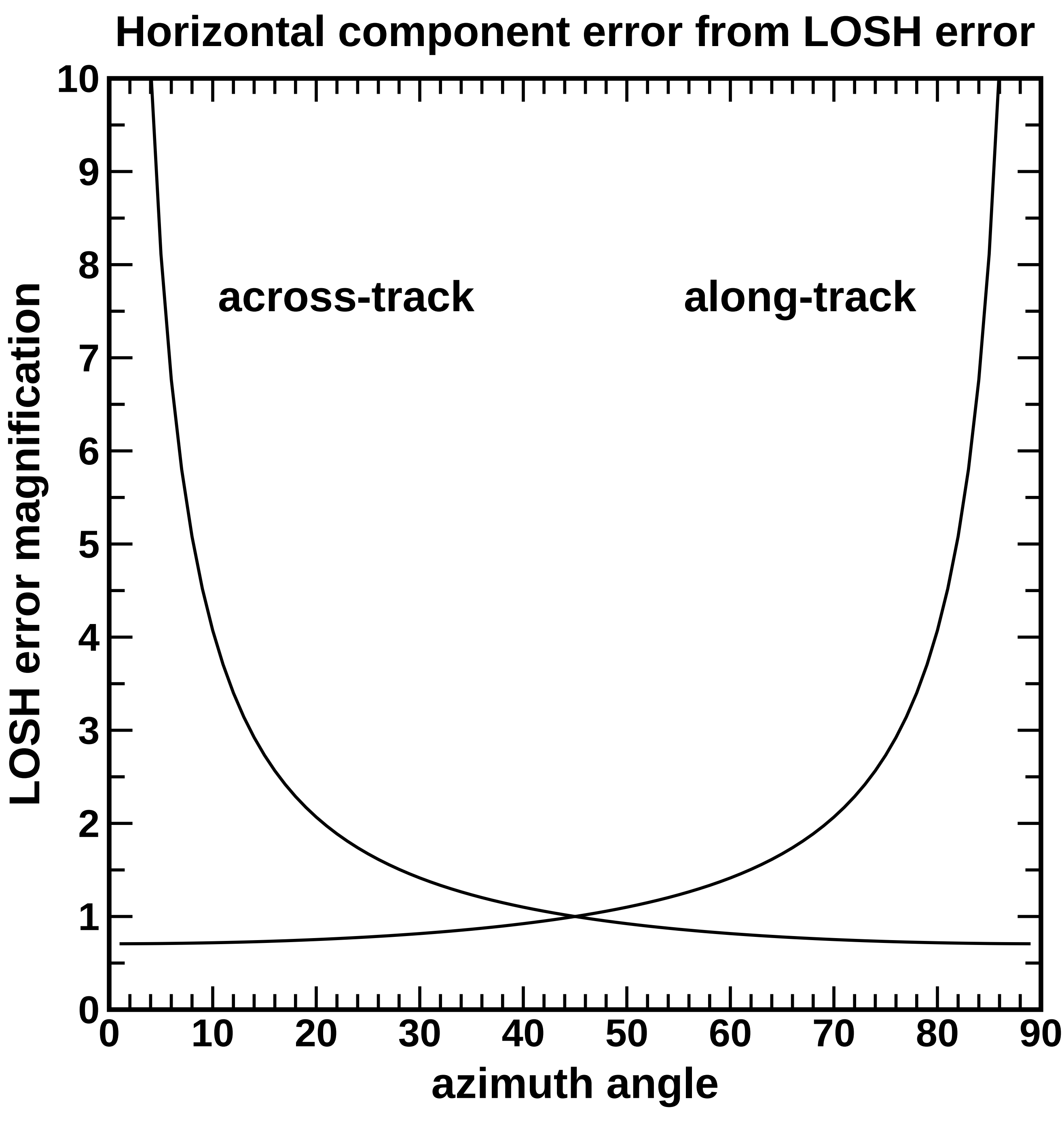
- **along-track random error component from σ_{LOS}**

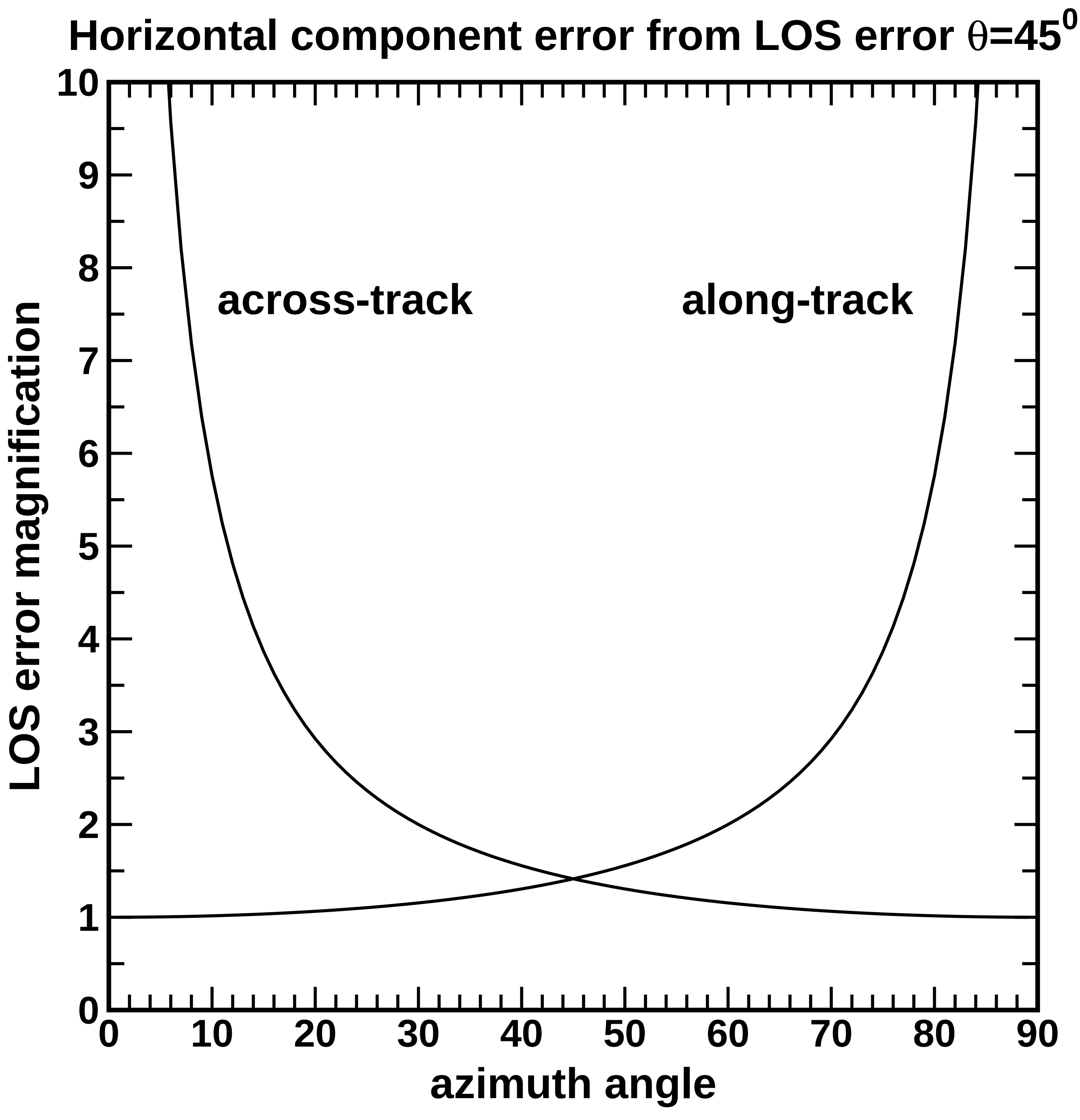
$$\sigma_u = \frac{\sigma_{LOS}}{\sqrt{2}\cos(\phi)\sin(\theta)} = \frac{\sigma_{LOSH}}{\sqrt{2}\cos(\phi)}$$

- **across-track random error component from σ_{LOS}**

$$\sigma_v = \frac{\sigma_{LOS}}{\sqrt{2}\sin(\phi)\sin(\theta)} = \frac{\sigma_{LOSH}}{\sqrt{2}\sin(\phi)}$$

- **random error depends on azimuth angle**


Horizontal component error from LOSH error
across-track
along-track
LOSH error magnification
azimuth angle
0
1
2
3
4
5
6
7
8
9
10
0
10
20
30
40
50
60
70
80
90



Horizontal component error from LOS error θ=45⁰
across-track
along-track
LOS error magnification
azimuth angle
0
1
2
3
4
5
6
7
8
9
10
0
10
20
30
40
50
60
70
80
90

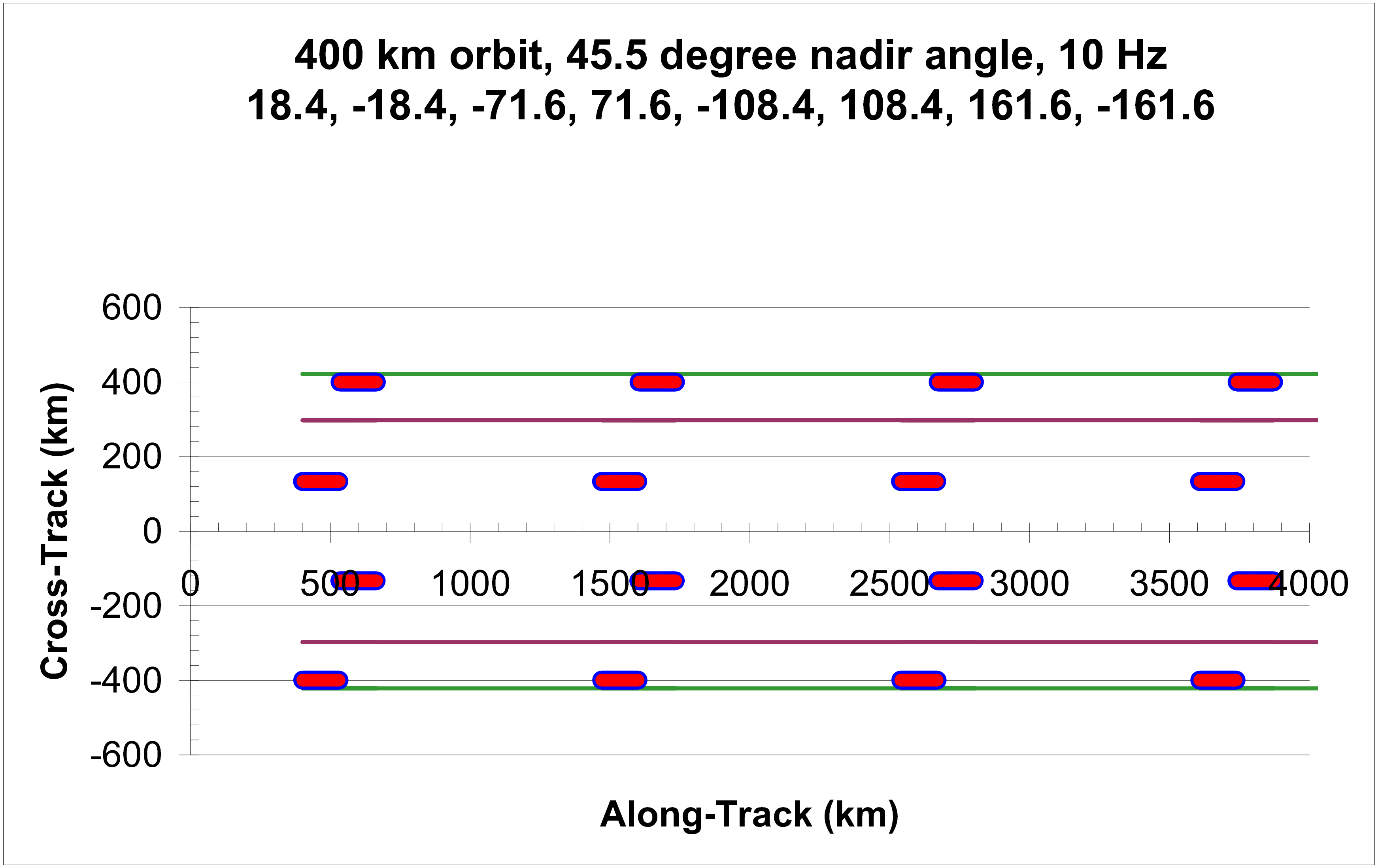
CRITERION FOR OPTIMAL LIDAR SCAN GEOMETRY

- **Overlaid for and aft tracks to sample the same atmosphere**

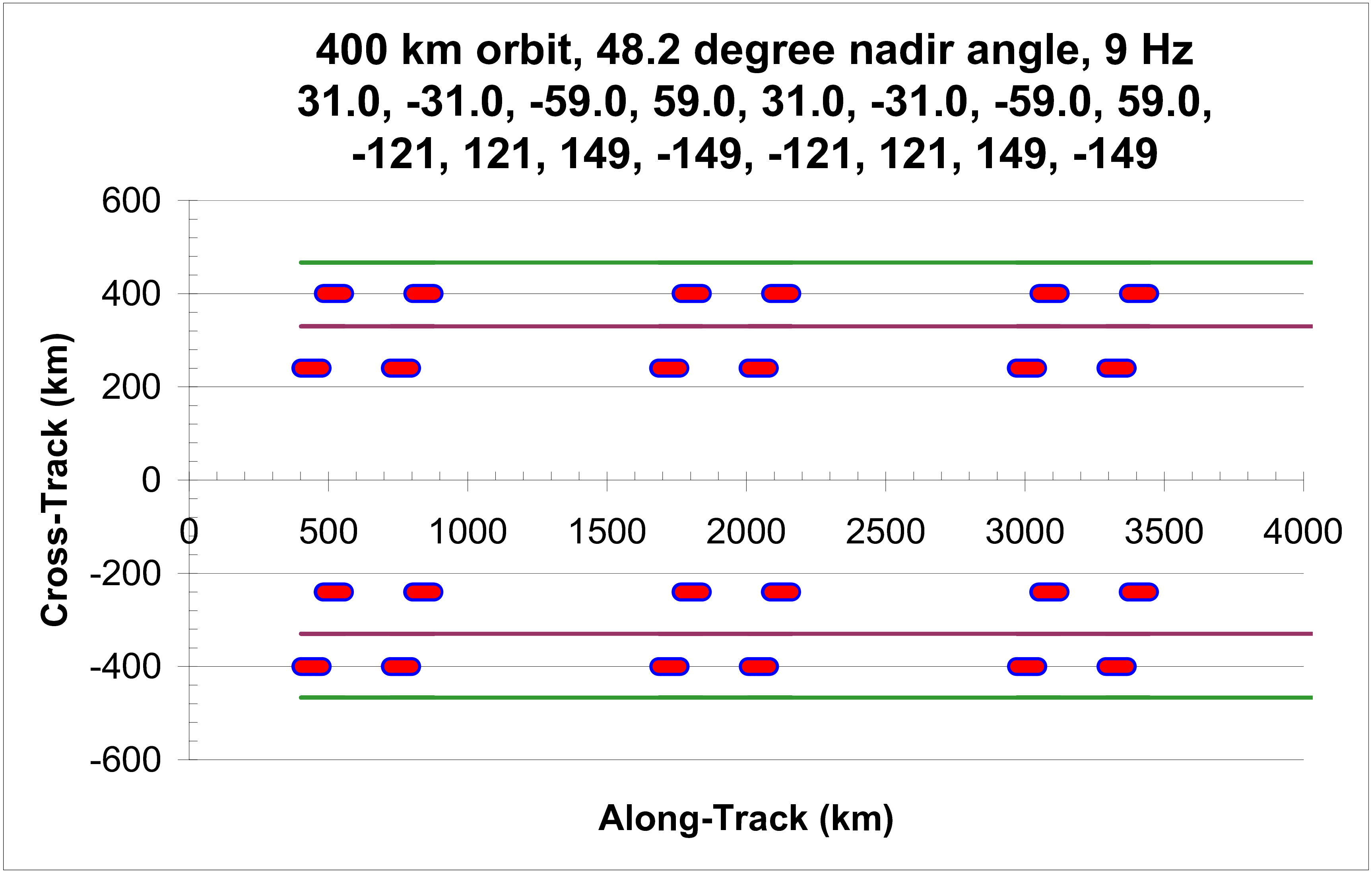
- **Require equal random error properties for each horizontal vector profile**

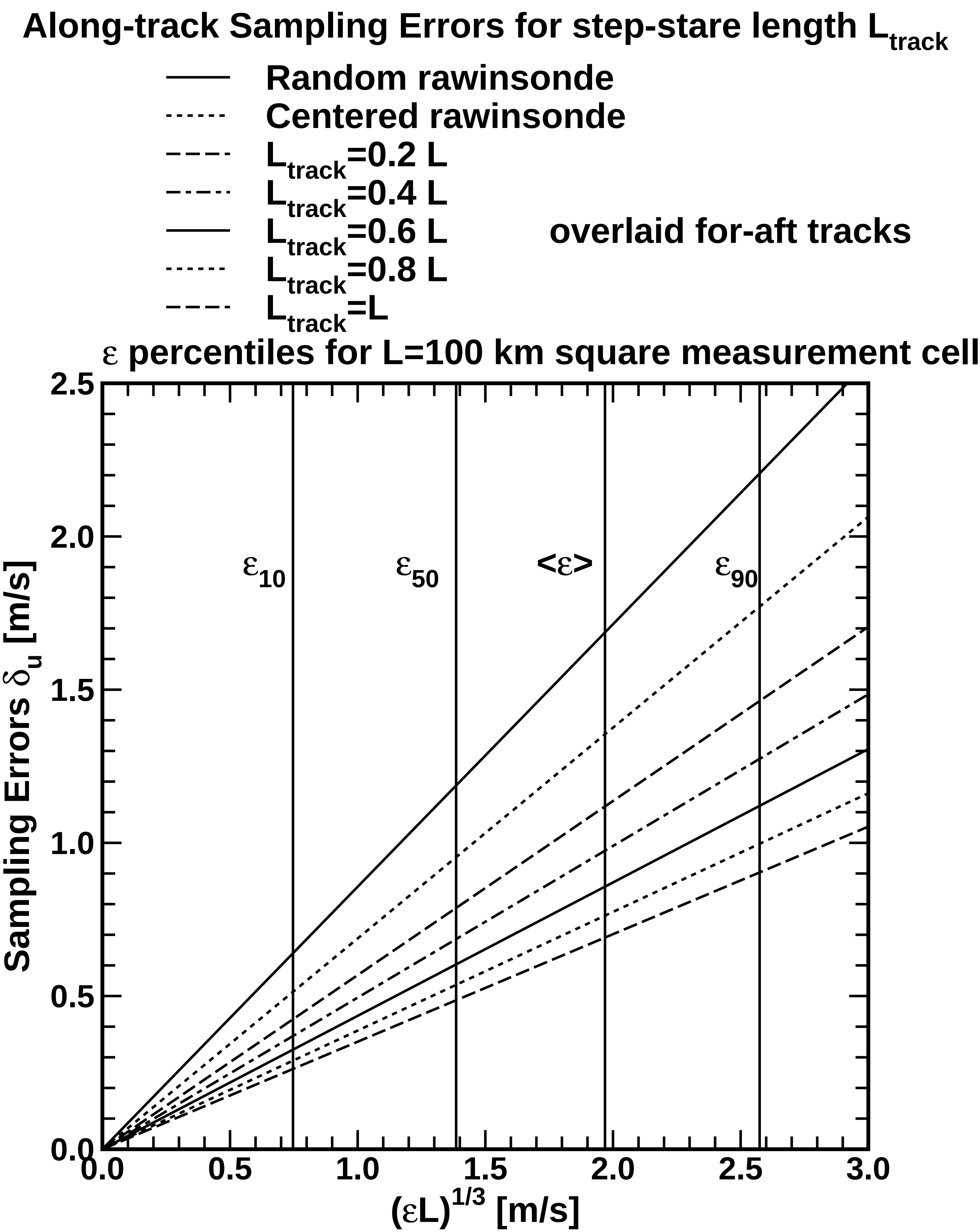
- **Mathematical solutions exist for flat earth and spherical earth**

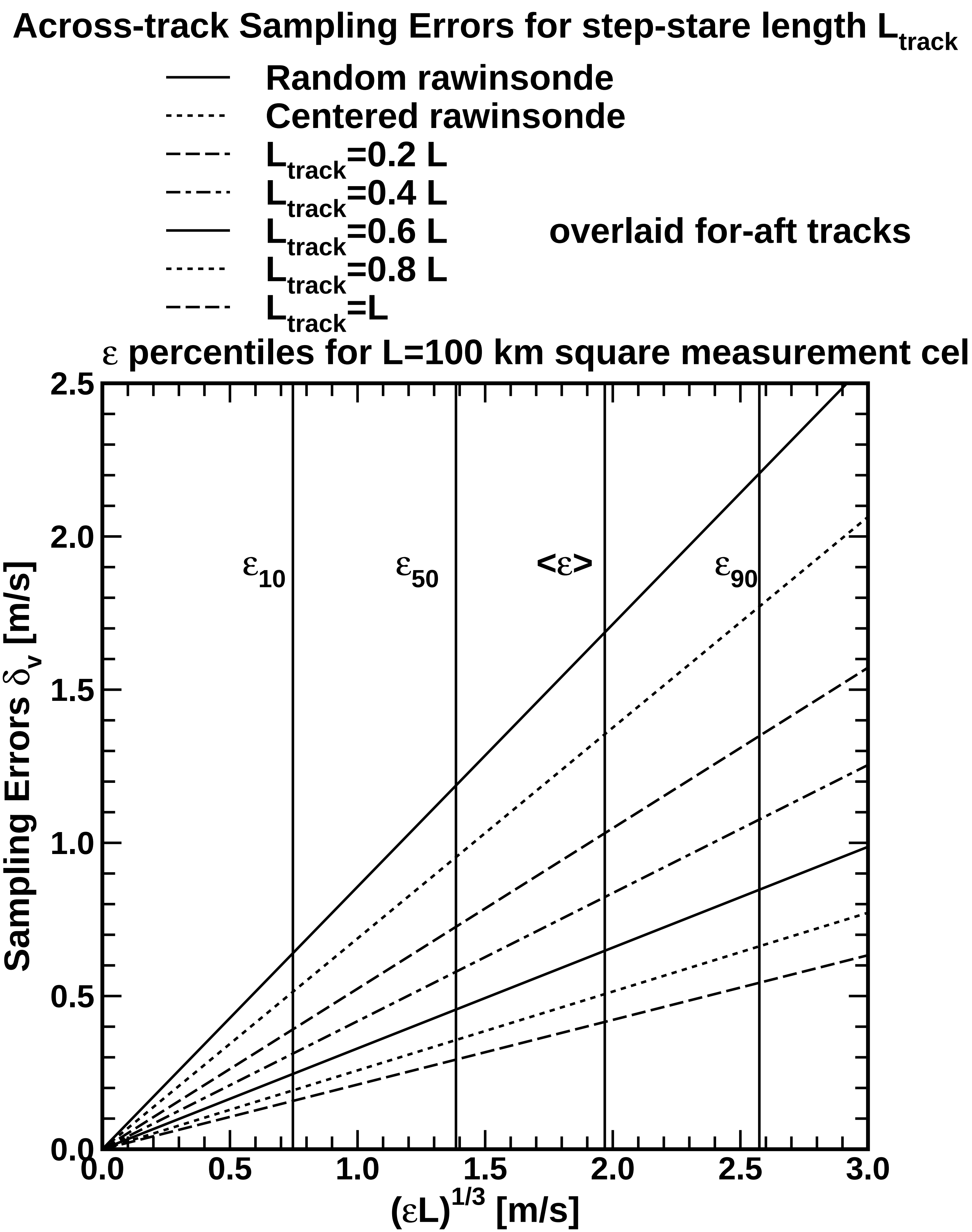
- **Trade-off between random error and measurement sampling geometry**

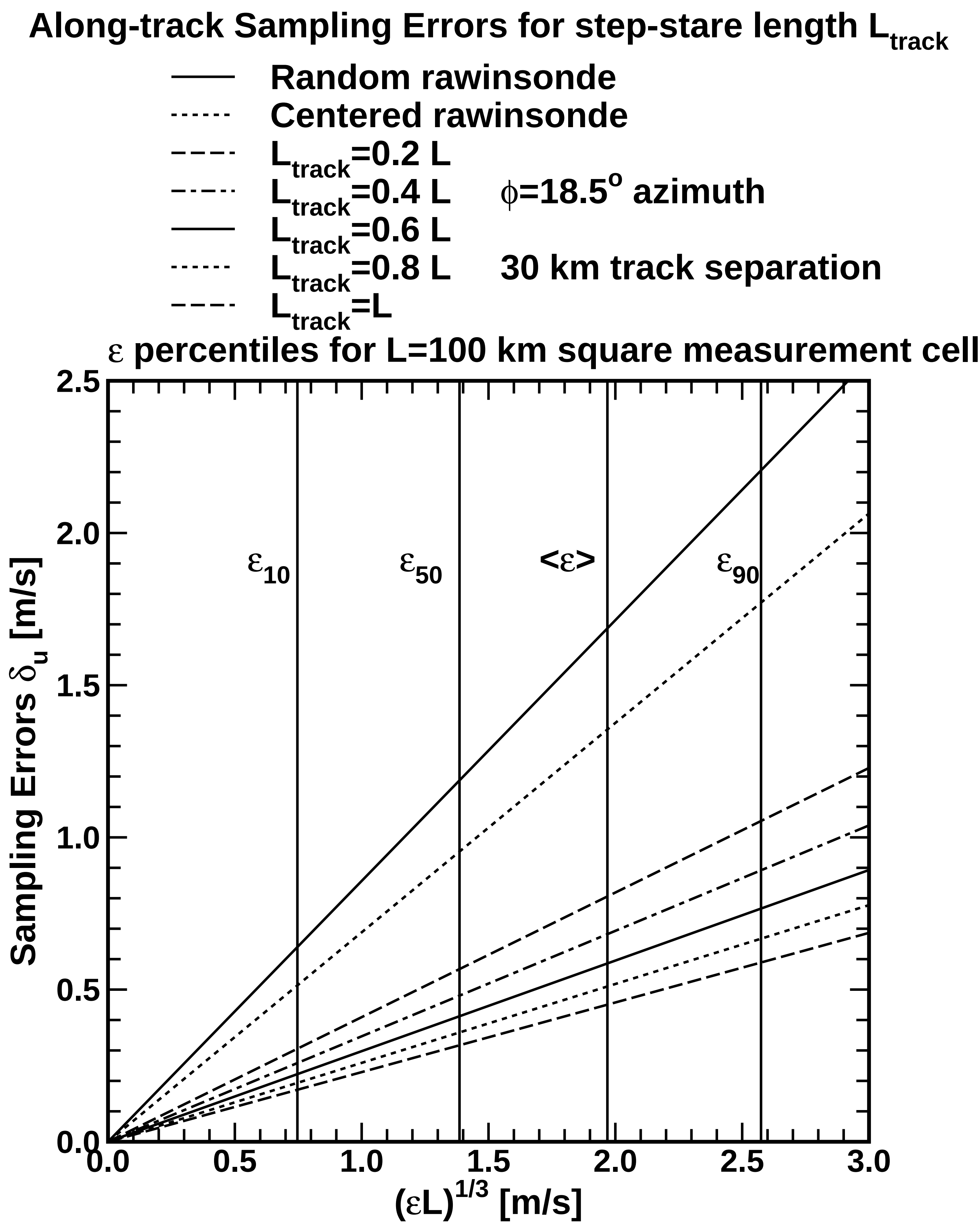


400 km orbit, 45.5 degree nadir angle, 10 Hz
18.4, -18.4, -71.6, 71.6, -108.4, 108.4, 161.6, -161.6
Cross-Track (km)
600
400
200
0
-200
-400
-600
0
500
1000
1500
2000
2500
3000
3500
4000
Along-Track (km)

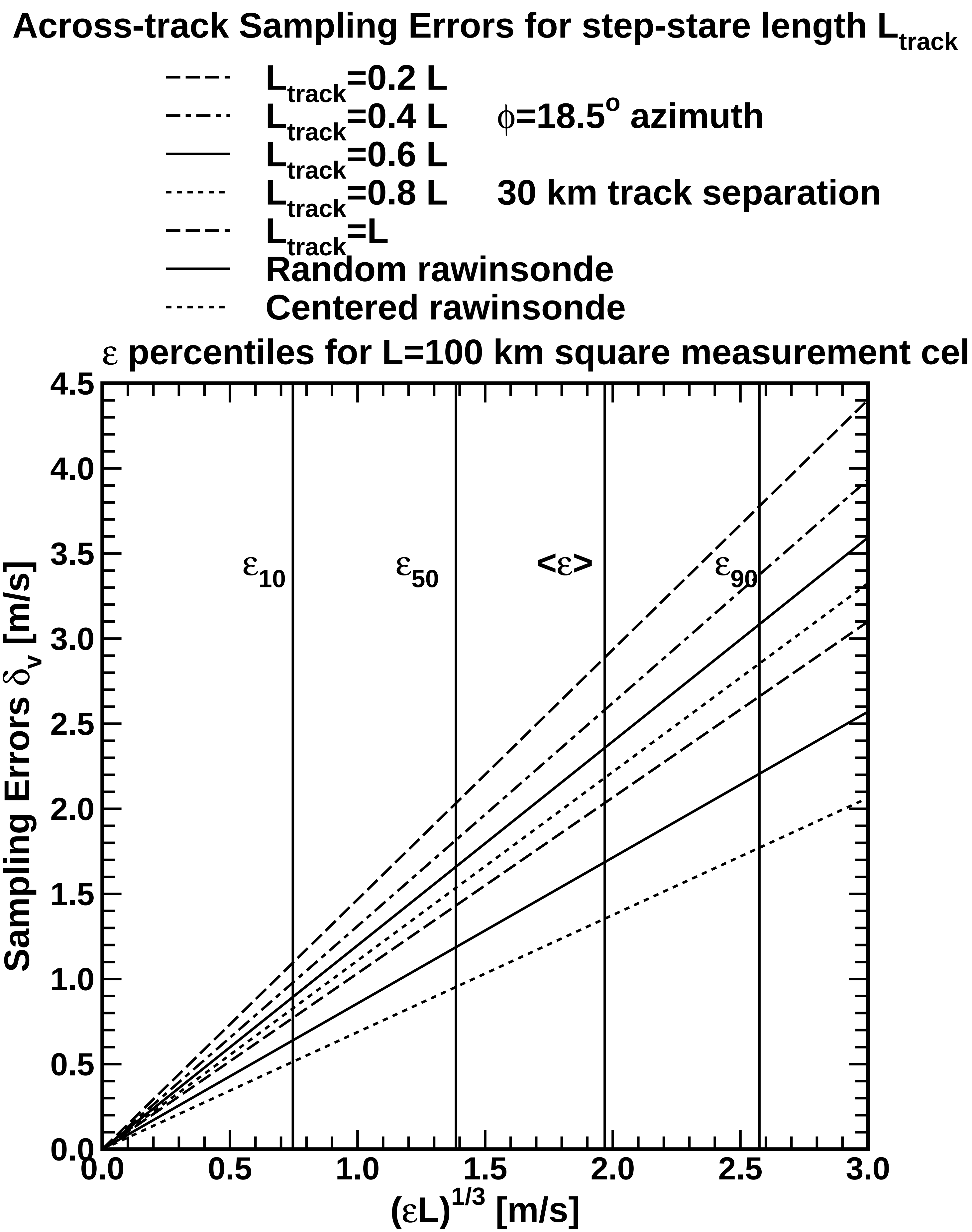

400 km orbit, 48.2 degree nadir angle, 9 Hz
31.0, -31.0, -59.0, 59.0, 31.0, -31.0, -59.0, 59.0,
-121, 121, 149, -149, -121, 121, 149, -149
600
400
200
0
-200
-400
-600
Cross-Track (km)
0
500
1000
1500
2000
2500
3000
3500
4000
Along-Track (km)


Along-track Sampling Errors for step-stare length L_{track}
Random rawinsonde
Centered rawinsonde
L_{track}=0.2 L
L_{track}=0.4 L
L_{track}=0.6 L
overlaid for-aft tracks
L_{track}=0.8 L
L_{track}=L
ε percentiles for L=100 km square measurement cell
ε_{10}
ε_{50}
$\langle\varepsilon\rangle$
ε_{90}
Sampling Errors δ_u [m/s]
$(\varepsilon L)^{1/3}$ [m/s]
0.0
0.5
1.0
1.5
2.0
2.5
3.0

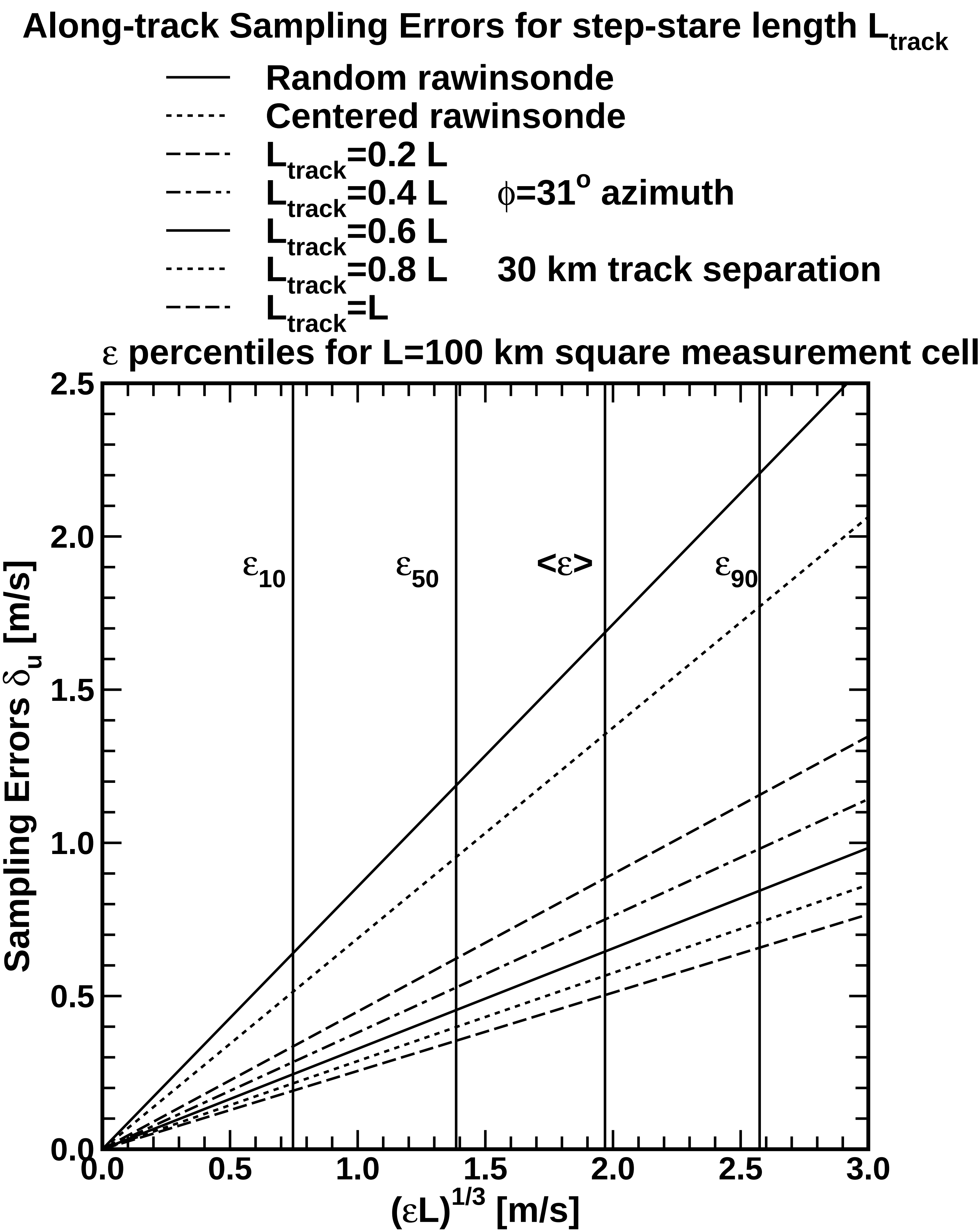

Across-track Sampling Errors for step-stare length L_{track}
Random rawinsonde
Centered rawinsonde
L_{track}=0.2 L
L_{track}=0.4 L
L_{track}=0.6 L
L_{track}=0.8 L
L_{track}=L
overlaid for-aft tracks
ε percentiles for L=100 km square measurement cell
ε_{10}
ε_{50}
<ε>
ε_{90}
Sampling Errors δ_v [m/s]
$(\varepsilon L)^{1/3}$ [m/s]
0.0
0.5
1.0
1.5
2.0
2.5
3.0



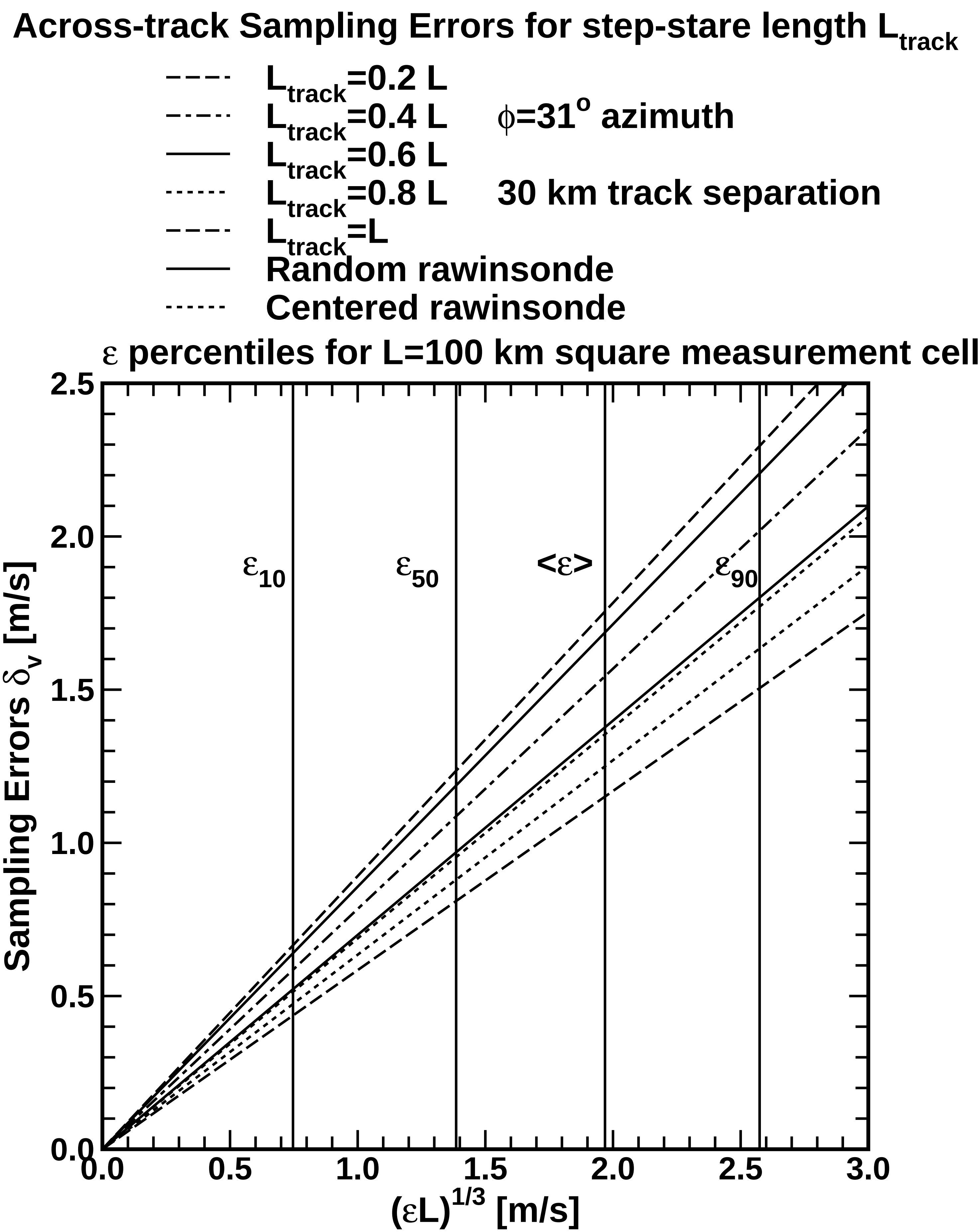
Along-track Sampling Errors for step-stare length L_track
Random rawinsonde
Centered rawinsonde
L_track=0.2 L
L_track=0.4 L
L_track=0.6 L
L_track=0.8 L
L_track=L
φ=18.5° azimuth
30 km track separation
ε percentiles for L=100 km square measurement cell
ε_10
ε_50
<ε>
ε_90
Sampling Errors δ_u [m/s]
(εL)^1/3 [m/s]
0.0
0.5
1.0
1.5
2.0
2.5
3.0



Across-track Sampling Errors for step-stare length L_track
L_track=0.2 L
L_track=0.4 L
φ=18.5° azimuth
L_track=0.6 L
L_track=0.8 L
30 km track separation
L_track=L
Random rawinsonde
Centered rawinsonde
ε percentiles for L=100 km square measurement cell
ε10
ε50
<ε>
ε90
Sampling Errors δv [m/s]
(εL)^1/3 [m/s]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5



Along-track Sampling Errors for step-stare length L_track
Random rawinsonde
Centered rawinsonde
L_track=0.2 L
L_track=0.4 L
φ=31° azimuth
L_track=0.6 L
L_track=0.8 L
30 km track separation
L_track=L
ε percentiles for L=100 km square measurement cell
ε10
ε50
<ε>
ε90
Sampling Errors δu [m/s]
(εL)^1/3 [m/s]
0.0
0.5
1.0
1.5
2.0
2.5
3.0



Across-track Sampling Errors for step-stare length L track
L track =0.2 L
L track =0.4 L
φ=31° azimuth
L track =0.6 L
L track =0.8 L
30 km track separation
L track =L
Random rawinsonde
Centered rawinsonde
ε percentiles for L=100 km square measurement cell
ε10
ε50
<ε>
ε90
Sampling Errors δv [m/s]
(εL)1/3 [m/s]
0.0
0.5
1.0
1.5
2.0
2.5
3.0

SUMMARY

- Critical trade-off between random errors and the location of the measurements

- Criterion for selecting optimal design

- Sampling error requires a statistical description of the velocity field

- Large sampling error and large random error from two LOS measurements with non-orthogonal perspectives

- Define data requirements in terms of the random error for each horizontal component or in terms of the random error of the wind speed

- Select priorities for the data requirements to guide the engineering trade-off