Radiative-convective models provide a reasonable representation of the globally averaged state of the surface-atmosphere system. They are much less reliable as analogues for the condition of the environment at any given latitude. Temperatures obtained on the basis of radiative-convective models for the tropics are too high, while temperatures calculated for temperate and higher latitudes are too low. The missing ingredient, as we saw in Section 7.7, relates to the influence of atmospheric motion, with an additional contribution due to transport of heat by the ocean. Significant quantities of heat are transferred by bulk motions of the atmosphere and of the ocean from low to high latitude, as was indicated, for example, in Figure 7.16. An understanding of climate presumes an understanding of the factors regulating the spatial redistribution of heat by atmospheric and oceanic motions.

Adjustment of the radiative state of the atmosphere, associated for example with changes in the concentration of a greenhouse gas such as CO$_2$, may be expected to alter not only the average temperature but also the patterns of circulation of the atmosphere. How might the climate change in response to such a disturbance? Radiative-convective models can provide, we hope, an indication of the change in temperature expected for Earth as a whole, but such predictions are of limited use to the policymaker seeking answers to more specific questions. In the presence of an enhanced concentration of CO$_2$, should we anticipate a shift in the position of Earth's major climatic or ecological zones, a change, for example, in the areal extent or geographic location of deserts? How might changes in climate affect the supply of fresh water, or the capacity of Earth to produce food? Should we expect an increase or decrease in the incidence of violent storms, in the frequency of hurricanes for example? To address these issues we need a physical model for the dynamics of the atmosphere and a complementary model for the ocean. At a minimum we need to understand the processes that regulate the transport of heat and moisture from one region of Earth to another, an appreciation for the factors that regulate the connections between climate and latitude.

This chapter concerns the physical mechanisms responsible for what is referred to as the general circulation of the atmosphere, defined for present purposes as the long-term average state of motion of the air. (The circulation of the ocean is discussed in Chapter 9.) The pattern of winds associated with the general circulation at the surface has been known, at least in broad outline, for almost three hundred years, ever since the golden age of exploration, when navigators such as James Cook (1728–1779) traveled the seven seas in search of adventure and trade, produced the first comprehensive outlines of our planet, and recorded (incidentally) the ever-fluctuating state of its weather. A conceptual model for longitudinally and time-averaged circulation is presented in Figure 8.1. According to this picture, winds in the tropics blow predominantly from the northeast in the Northern Hemisphere and from the southeast in the Southern Hemisphere. Winds at midlatitudes (30°–60°) blow mainly from the west or southwest at midlatitudes in the Northern Hemisphere, from the west or northwest at comparable latitudes.
in the Southern Hemisphere. Winds at high latitudes (60°–90°) carry air from polar regions to lower latitudes, from a generally easterly direction in both hemispheres.\(^1\)

The system of easterly winds in the tropics played an important role in early navigation, providing fast and reliable assistance to ships sailing from Europe to the New World. Merchants referred to the helpful winds on the outward trip as the “Trade Winds.” The favored path for the return journey, despite dangers and the often devastating tolls taken by storms, was to the north, where the prevailing winds were from the west. Separating the zone of easterlies from westerlies was a region where winds were generally slight and temperatures unrelentingly hot under cloudless skies, where it was not uncommon for ships to be becalmed for days or even weeks. Samuel Taylor Coleridge (1772–1834) described the region graphically in his “Rime of the Ancient Mariner”:

All in a hot and copper sky,
The bloody Sun, at noon,
Right up above the mast did stand,
No bigger than the Moon.
Day after day, day after day,
We stuck, nor breath nor motion;
As idle as a painted ship
Upon a painted ocean.
Water, water, every where,
And all the boards did shrink;
Water, water, everywhere,
Nor any drop to drink.

This area is known as the Horse Latitudes (around 30° both north and south), reputedly because horses were often killed on ships in this region to provide food and conserve water.

The picture of the general circulation displayed in Figure 8.1 is at best schematic. The actual situation is considerably more complicated, as indicated by maps of surface winds for the months of January and July, presented in Figures 8.2 and 8.3. The results given here are intended to illustrate the pattern of the surface flow expected when winds are averaged over many years for the months of January and July. The results demonstrate the importance of high and low pressure systems in guiding the motion of air, particularly at mid and high latitudes. Air in the Northern Hemisphere tends to circulate clockwise around regions of high pressure, in the opposite direction around zones of low pressure. (Reasons for this behavior are discussed below.) The pattern is reversed for the Southern Hemisphere. The highs and lows in Figures 8.2 and 8.3 are associated with particular geographic features. They tend to remain relatively fixed in position over the course of a season. The high pressure system over the Soviet Union in January, for example, is caused by the intense cooling and consequent sinking of air over the Asian continent during winter. The analogous lows over the Aleutian Islands and Iceland are associated with the relatively warm waters of the North Pacific and Atlantic.

A map of surface pressure for any given day in January or July, indeed for any time of year, would reveal high and low pressure systems in addition to the semipermanent features apparent in Figures 8.2 and 8.3. Unlike the features in the figures, these systems are mobile and dynamic. They appear and disappear over the course of a few days and can travel vast distances in the brief period between their births and decay. Averaged over the course of a month, disturbances associated with mobile highs and lows tend to cancel at any given location, leaving minimal traces of their presence on long-term average charts such as those displayed in Figures 8.2 and 8.3. In combination with stationary systems, however, as we shall see, they play an important role in the global transport of heat and are responsible for much of the weather—the day-to-day variable manifestations of climate—experienced at mid and high latitudes.\(^2\)

The general circulation defines the complex response of the atmosphere to unequal rates at which solar radiation is absorbed over time and position and to related imbalances in rates at which energy is emitted to space in the infrared portion of the spectrum. As previously noted, there is a surplus of radiant energy (visible plus infrared) in the tropics, with a corresponding deficit at higher latitudes. Imbalances can also arise on much smaller spatial scales, between land areas and the surrounding ocean, for example.

We begin our account of the dynamics of the atmosphere in Section 8.1 with a qualitative description of the circulation arising near the sea in summer as a consequence of differences in the responses of land and water to insolation (exposure to the Sun’s rays) over the course of a day. These differences give rise, as we shall see, to a shallow circulation (one confined to low altitude) with on-shore breezes at the surface during the day and off-shore breezes at night. The sea (or land) breeze problem provides a useful introduction
to the circulation of the global atmosphere driven by differences in the absorption of heat at low as compared to high latitudes. An early model for the general circulation of the atmosphere is presented in Section 8.2. The role of rotation and its effect on atmospheric motion is discussed in Sections 8.3–8.6. We introduce in Sections 8.3 and 8.4 the Coriolis force, and in Section 8.6 the concept of geostrophy, in which the horizontal gradient of pressure acting on an air parcel is balanced primarily by the Coriolis force. The role of friction is discussed in Section 8.7. Further perspectives on the circulation of the atmosphere are presented in Section 8.8 as a prelude to a discussion in Sections 8.9–8.11 of the angular momentum budget of the atmosphere and the importance of transport by eddies. Summary remarks are presented in Section 8.12.

8.1 The Land-Sea Breeze Problem

Imagine a situation in which weather conditions near the land-sea boundary are initially uniform: there is no significant variation in the horizontal field of atmospheric pressure. Over land, solar energy is absorbed only by the surface. When absorbed by the sea, however, solar energy is distributed over a depth of several meters. Thus land temperature rises rapidly during the day and cools equally quickly at night, while ocean temperature responds more slowly, staying relatively constant over the course of a day.

As the temperature rises at the surface of the land during the day, the temperature of the air above the ground rises, as well. The pressure at the surface, determined by the mass of the overlying atmosphere, initially remains constant. Pressure, temperature, and density are constrained to satisfy the perfect gas law (equation 7.7). With pressure fixed, the increase in temperature at ground level over land is compensated by a decrease in density. To accommodate the decrease in density, air expands and the atmosphere rises at every level above the surface. This establishes a pressure gradient in the horizontal direction above the surface: pressure is higher at any given level over the warm ground than over the sea.

The variation of pressure with altitude may be calculated using the barometric law, (equation 7.15). The scale height, given by equation (7.11), is larger over land during
the day than over the sea, as a consequence of the higher temperature. The difference in the rate at which pressure drops with altitude over land and sea is illustrated schematically in Figure 8.4.

Example 8.1: What is the difference in pressure between two parcels of air at 500 m if one parcel, over land, is at 300 K and the other parcel, over the sea, is at 285 K? Assume that the temperature over the altitude range 0–500 m is the same as the value at the surface.

Answer: First find the appropriate scale heights over land (l) and sea (s):

\[ H_l = \frac{RT_l}{g} = \frac{(2.87 \times 10^6 \text{ erg g}^{-1} \text{ K}^{-1})(300 \text{ K})}{980 \text{ cm sec}^{-2}} = 8.79 \times 10^3 \text{ cm} \]

\[ H_s = \frac{RT_s}{g} = \frac{(2.87 \times 10^6 \text{ erg g}^{-1} \text{ K}^{-1})(285 \text{ K})}{980 \text{ cm sec}^{-2}} = 8.35 \times 10^3 \text{ cm} \]

Now use the scale heights to find the two pressures.

Pressure aloft = (surface pressure) \( \left( \exp \left( \frac{-\text{height}}{\text{scale ht}} \right) \right) \)

\[ P_l(500\text{m}) = 10^6 \text{ dyn cm}^{-2} \exp \left[ \frac{-0.5 \text{ km}}{8.79 \text{ km}} \right] \]
\[ = 9.45 \times 10^5 \text{ dyn cm}^{-2} \]

\[ P_s(500\text{m}) = 10^6 \text{ dyn cm}^{-2} \exp \left[ \frac{-0.5 \text{ km}}{8.35 \text{ km}} \right] \]
\[ = 9.42 \times 10^5 \text{ dyn cm}^{-2} \]

\[ \Delta P = P_l - P_s = 9.45 \times 10^5 \text{ dyn cm}^{-2} - 9.42 \times 10^5 \text{ dyn cm}^{-2} \]
\[ = 3.0 \times 10^3 \text{ dyn cm}^{-2} \]

An air parcel at any given height above the surface in the transition zone between land and sea experiences a net force pushing it toward the sea during a summer day, as illustrated in Figure 8.5. The magnitude of this force de-
The Land-Sea Breeze Problem

Figure 8.4 Schematic illustration of the variation of pressure with altitude. Curve A applies to the warmer conditions characteristic of air over land in the summer. Curve B applies to colder air over the ocean.

Figure 8.5 Airflow near the seashore during the day in summer.

Figure 8.6 Airflow near the seashore at night when air over the sea is warmer than air over land. Compare the pressure profiles to those in Figure 8.5.

east and the Gulf of Mexico to the west, as indicated in Figure 8.7. Air drawn from off the sea has a high-moisture content. As it moves over land, converging from both west and east, it rises, setting off a bank of thunderstorms frequently extending along the entire axis of the state during early afternoon. A vertical perspective on the circulation, a cut through the atmosphere from west to east, is presented in Figure 8.8. Convergence of surface air and the subsequent rising motion over land can give rise to heavy daytime thundershowers.

Figure 8.7 Pattern of onshore surface winds for Florida during summer.

Figure 8.8 Vertical west-east cross section of the daytime circulation during summer in Florida. Rising air over land is supplied with abundant H₂O from the surrounding ocean, resulting in frequent daytime thundershowers.
8.2 Hadley’s Model for the General Circulation

The first serious attempt to account for the general circulation of the atmosphere was presented by the English meteorologist George Hadley in 1735. Hadley began his paper with a bold statement: “I think the causes of the General Trade-Winds have not been fully explained by any of those who have wrote on that Subject.” Building on an earlier paper by the astronomer Edmund Halley (1686), who presented a detailed description of the trade winds observed in three oceans, Hadley sought to explain the westerly winds at the surface at mid-latitudes and the easterlies at low latitudes as natural consequences of the preferential absorption of solar radiation near the equator. He envisaged global circulation as a large-scale analogue to the sea breeze circulation outlined above. He assumed that air would rise over the tropics, split toward the north and south aloft, and return to the surface at higher latitudes in both hemispheres. The circulation would be completed by a return flow to the tropics at the surface in both hemispheres, as illustrated in Figure 8.9.

Hadley supposed that air, as it moved, would tend to conserve its absolute speed; in other words, air speed, as measured by an observer fixed in space, would appear to remain constant. The conservation of absolute speed is not a physical law, and, as we shall see, this assumption led to erroneous conclusions. However, the contrast between rotating and inertial frames of reference did lead to important ideas about circulation worth exploring here in some detail.

Recall that points on Earth’s surface are not fixed in space but are always in a state of rapid motion, from west to east, as a consequence of planetary rotation. Speeds of rotation are largest at the equator: a body fixed to Earth at the equator moves a distance equal to the circumference of Earth, $2\pi$ times the radius of the planet $(6.378 \times 10^6 \text{ cm})$, over the course of a day $(8.64 \times 10^4 \text{ sec})$, corresponding to a speed, as measured by a space-fixed observer, of

$$v_r = \frac{2(3.14)(6.378 \times 10^6 \text{ cm})}{8.64 \times 10^4 \text{ sec}} = 4.64 \times 10^4 \text{ cm sec}^{-1} \quad (8.1)$$

Here the numerator defines the distance moved in a day, or the circumference; the denominator is equal to the length of a day measured in seconds. Imagine touching a magic marker to a spinning globe. The circle traced at the equator is larger than the circle that would be traced at any other latitude. Figure 8.10 should convince you that the distance traveled in a day by a fixed point on Earth is $2\pi R(\cos \Lambda)$, where $\Lambda$ represents the latitude. Magnitudes of rotational velocities for different latitudes are given in Table 8.1 as $v_r$.

Consider an air parcel, initially stationary with respect to Earth’s equator, that begins to move northward. Following Hadley, assume that the absolute speed of the parcel remains constant, meaning equal to the rotation speed of the planet at the equator, or $4.6 \times 10^4 \text{ cm sec}^{-1}$ measured in the west-east direction. As the parcel moves northward, it begins to drift eastward: its eastward velocity is larger than the velocity of the solid Earth below. An observer on Earth would interpret the parcel’s relative motion as a westerly wind. The speed of the wind at latitude $\Lambda$ would be given by the difference between the rotation speed of Earth at the equator and the rotation speed at the local latitude. Values of the wind speed computed in this fashion are included in Table 8.1 as $v$.

---

**Example 8.2:** Calculate the expected wind speed (as perceived by an observer rotating with Earth) based on Hadley’s concept of constant absolute speed for a parcel moving from the equator to $60^\circ \text{N}$ or S.

**Answer:** First, calculate the absolute speed of the parcel stationary at the equator. As shown in equation (8.1), it equals $4.6 \times 10^4 \text{ cm sec}^{-1}$.

Next, calculate the absolute speed of a parcel stationary at $60^\circ \text{N}$ or S.

$$v_s = \frac{2(3.14)(6.378 \times 10^6 \text{ cm})(\cos 60^\circ)}{8.64 \times 10^4 \text{ sec}}$$

$$= 2.3 \times 10^4 \text{ cm sec}^{-1}$$

Finally, find the difference between the rotation speed at the equator and that at $60^\circ \text{N}$ or S.

$$(4.6 \times 10^4) - (2.3 \times 10^4) = 2.3 \times 10^4 \text{ cm sec}^{-1}$$

This is over 800 km per hour! □

As seen in this example, air reaching high latitudes in Hadley’s scheme would have extremely strong flows eastward, and, in the absence of friction, the winds at the surface would be a mirror image of the flow aloft. As the air returns to the equator at the surface, the strength of the westerly wind would decrease steadily as it moves to
lower latitudes. In this case, the model would account for
the presence of westerly winds at midlatitudes. It would
fail, however, to address the primary problem of interest
to Hadley: the existence of easterly trade winds in the
tropics.

Hadley resolved the dilemma by noting that friction in-
duced by transfer of momentum from air to ground
would be expected to reduce the magnitude of the surface
wind speed. He suggested that friction could cause the speed
of the surface westerly wind to drop to zero at a latitude of
about 30°. Prevailing winds at lower latitude would switch
then to an easterly direction, reflecting the inability of air on
its return journey to the equator to keep up with the in-
creasing speed associated with west-east rotation of the
underlying planet. A schematic illustration of Hadley’s cir-
culation is presented in Figure 8.11.

Hadley’s contribution to our understanding of the gen-
eral circulation of the atmosphere is remarkable, especially
when we recall that his paper appeared more than 250 years
ago. He offered the first detailed description of circulation
arising as a result of differential heating. He correctly pointed
to the importance of friction in limiting the magnitude of
wind velocities near the surface. He provided the first plau-
sible physical explanation for the general features of the circu-
lation observed not only at the surface but also aloft. His
paper, predating observations of the winds aloft, successfully
predicted at least the eastward direction of the upper level
winds. In this sense, Hadley may be credited with the first
successful weather forecast!

His paper may be criticized on grounds that it failed
to account for complications associated with the presence
of continents and oceans and that it did not allow for sea-
sonal variations in the absorption of solar radiation. A
more serious error, perhaps, was his assumption that air
should tend to conserve the magnitude of its absolute ve-
locity. It would have been preferable to suppose that
motion should be constrained to conserve angular mo-
momentum (see Section 2.10). Conservation of angular
momentum, as previously noted, requires that the rota-
tional speed of a skater increase as the arms are drawn in.
For the same reason, the absolute value of the west-east
speed of an air parcel should increase, rather than stay
constant, as the parcel approaches the axis of rotation of
the planet in moving to higher latitude. Hadley’s model
can be corrected in a relatively straightforward way to
allow for conservation of angular momentum rather than
absolute velocity. As we shall show, constraints on the
west-east velocity imposed by the tendency of motion to
conserve angular momentum may be treated by introduc-
ing a new effective force, the Coriolis force, named in
honor of the nineteenth-century French physicist Gaspard
Coriolis (1792–1843).
8.3 Angular Momentum and the West-East Coriolis Force

Denote the absolute value of the speed of an air parcel in the west-east direction at latitude \( \lambda \) by \( v_a(\lambda) \). Suppose that the mass of the air parcel is equal to 1 gram. The magnitude of the angular momentum vector is directed to the north, aligned with the axis of rotation of the earth. The angular momentum of the parcel at latitude \( \lambda \), \( L(\lambda) \), is given by

\[
L(\lambda) = (v_a(\lambda)) R \cos \lambda \tag{8.2}
\]

With \( v_a(\lambda) \) expressed in cm sec\(^{-1}\) and the radius of Earth, \( R \), in cm, \( L \) has units g cm\(^2\) sec\(^{-1}\) (remember, we are considering a parcel with a mass of 1 g, as we saw in Chapter 2, the magnitude of the angular momentum vector is proportional to the mass of the parcel). Here \( R \cos \lambda \) defines the distance separating the parcel from Earth's rotational axis at latitude \( \lambda \) (see Figure 8.10).

Imagine that the parcel is initially at rest with respect to Earth's equator. The absolute value of the west-east speed is given by in this case

\[
v_a(0) = \frac{2\pi R}{(8.64 \times 10^4 \text{ sec})} \tag{8.3}
\]

Note that this equation is the same as (8.1). However, we indicate the velocity of the particle here by \( v_a(0) \) rather than \( v_r \) to emphasize that we are dealing with the absolute wind speed rather than the speed relative to motion of the solid body; the air parcel is considered as though it were attached to the ground below it, meaning it is assumed to be stationary with respect to Earth. With \( R \) given in centimeters \( (6.378 \times 10^8 \text{ cm}) \), the speed calculated using (8.3) has units of cm sec\(^{-1}\).

As discussed in Chapter 2, the quantity \( 2\pi R / (8.64 \times 10^4 \text{ sec}) \) in (8.3) is known as the angular velocity, written as \( \Omega \). It defines the rate at which the angular position of a particle changes with time, as illustrated in Figure 8.12. The angular velocity for Earth is equal to \( 7.3 \times 10^{-5} \text{ sec}^{-1} \). As indicated by equation (8.3) (see also Section 2.11), the speed corresponding to solid body rotation at the equator is given by

\[
v_a(0) = \Omega R \tag{8.4}
\]

At every other latitude, the solid body rotation is slower, with speeds given (see \( v_r \) in Table 8.1) by

\[
v_a(\lambda) = \Omega R \cos \lambda \tag{8.5}
\]

\( R \cos \lambda \) is always less than or equal to \( R \), and as \( \lambda \) approaches \( 90^\circ \) (the poles), \( \cos \lambda \) approaches zero. Hence, the speed for solid body rotation decreases from the equator to the poles.

Example 8.3: Find the angular momentum of an air parcel, with a mass of 1 g at rest with respect to Earth at the equator (latitude \( 0^\circ \)).

Answer: Recall that

\[
L(\lambda) = v_a(\lambda) R \cos \lambda \times 1 \text{ g} \tag{8.2}
\]

Then

\[
L(0) = v_a(0) R \cos(0) (1 \text{ g}) \tag{8.6}
\]

\[
= v_a(0) R (1 \text{ g}) \tag{8.7}
\]

\[
L(0) = \Omega R^2 (1 \text{ g})
\]

\[
= (7.3 \times 10^{-5} \text{ sec}^{-1})(6.38 \times 10^8 \text{ cm})^2 (1 \text{ g})
\]

\[
= 2.97 \times 10^{13} \text{ g cm}^2 \text{ sec}^{-1},
\]

where the subscript \( r \) is included to emphasize that the quantity refers to an air parcel stationary with respect to Earth (or, solid body rotation).

Suppose that the air parcel with a mass of 1 g, which we assumed to be initially at rest with respect to Earth's equator, begins to move northward, as would be the case for air in the upper branch of the Hadley circulation. Its angular momentum at the equator would be given by equation (8.7). For conservation of angular momentum to hold, the value of \( L \) at \( \lambda \) must equal the initial value appropriate for solid-body rotation at the equator.

Thus, the air parcel at other latitudes must develop a velocity with respect to Earth. Using the value for \( v_r(0) \) in (8.4) and equating (8.2) with (8.7), we find,

\[
L(\text{latitude } \lambda) = L(\text{equator})
\]

\[
v_a(\lambda) R \cos \lambda = \Omega R^2 \tag{8.8}
\]

\[
v_a(\lambda) = \frac{\Omega R}{\cos \lambda} \tag{8.9}
\]

---

![Figure 8.12](image-url)  
Change in the angular position of a rotating parcel with time. For a parcel rotating at angular velocity \( \Omega \), the angle \( \theta \) traced out in time \( t \) is equal to \( \Omega t \).
Values of \( v_o(\lambda) \) are given for different latitudes in Table 8.2. This value of \( v_o(\lambda) \) reflects a combination of Earth's velocity and the air parcel's velocity with respect to Earth. Table 8.2 also includes values for the corresponding westerly wind speed (the magnitude of the west-east velocity as measured by an observer fixed with respect to Earth), denoted simply as \( v(\lambda) \), obtained by subtracting from \( v_o(\lambda) \) the speed, \( v_i(\lambda) \), associated with planetary rotation given by (8.5):

\[
v(\lambda) = v_o(\lambda) - v_i(\lambda)
\]

(8.10)

\[
v(\lambda) = \frac{\Omega R}{\cos \lambda} - \Omega R \cos \lambda
\]

(8.11)

The results in Table 8.2 indicate that an air parcel moving northward from the equator and conserving angular momentum will tend to pick up speed eastward as it moves to higher latitudes. Viewed by an observer on Earth, the particle appears to turn to the right of its direction of motion. According to Newton's law of motion, equation (2.1), a change in velocity implies an acceleration and must be accompanied by a force. There is no obvious force we can point to as responsible for the apparent eastward drift of the northward-moving air parcel. Fortunately, there is a straightforward trick that can be used to account for the complications arising for motion in a rotating frame. This is done by allowing for an additional force, the Coriolis force mentioned above. The simple expression of Newton's law of motion for an inertial frame is preserved for a rotating frame, when we allow for effects of the Coriolis force. The following discussion is intended to elucidate the nature and form of the Coriolis force.

As we have seen, an air parcel moving northward from the equator appears to turn eastward. Similarly, an air parcel moving southward toward the equator would appear to reverse the path followed by the particle moving northward; it would turn westward. The effect of rotation in both cases is to cause the air parcel to turn to the right with respect to its direction of motion: eastward for the particle moving northward, westward for the particle moving southward. This defines the first important property of the west-east Coriolis force: it acts to the right of the direction of motion in the Northern Hemisphere.

We can apply the same argument for meridional (north-south) motion in the Southern Hemisphere. An air mass moving southward from the equator acquires a component of eastward velocity with respect to Earth; it turns to the left with respect to its direction of motion. An air parcel moving northward in the Southern Hemisphere drifts to the west; again it appears to turn to the left. The Coriolis force in the zonal (west-east) direction acts to the right of the direction of motion in the Northern Hemisphere, to the left in the Southern.

Consider an air parcel traveling southward across the equator, moving from the Northern to the Southern Hemisphere. The Coriolis force on the parcel acts westward in the Northern Hemisphere, eastward in the Southern. There is no reason to expect an abrupt change in the force as the parcel crosses the equator. In fact, there is a smooth variation in the magnitude and direction of the force with latitude. The change in direction crossing the equator occurs because the west-east Coriolis force vanishes at the equator; its magnitude there exactly equals zero.

Now consider how we might expect the zonal (west-east) component of the Coriolis force to vary with meridional (south to north) velocity. If the magnitude of the absolute west-east speed at latitude \( \lambda_1 \) is given by \( v_o(\lambda_1) \) and if angular momentum is conserved, the absolute value of the speed at latitude \( \lambda_2 \) may be obtained by equating the values for angular momentum at the two latitudes using the appropriate forms of equation (8.11):

\[
L(\lambda_1) = L(\lambda_2)
\]

(8.12)

\[
v_o(\lambda_1) R \cos \lambda_1 = v_o(\lambda_2) R \cos \lambda_2
\]

(8.13)

It follows that

\[
v_o(\lambda_2) = v_o(\lambda_1) \frac{\cos \lambda_1}{\cos \lambda_2}
\]

(8.14)

Using the expression for the solid body rotation speed given by equation (8.5), equation (8.14) can be rewritten to yield an expression relating the zonal wind speed at \( \lambda_2 \) to the corresponding quantity at \( \lambda_1 \). First, rewrite equation (8.10)

\[
v_o(\lambda) = v(\lambda) + \Omega R \cos \lambda
\]

and substitute into equation (8.14)

\[
v(\lambda_2) + \Omega R \cos \lambda_2 = [v(\lambda_1) + \Omega R \cos \lambda_1] \frac{\cos \lambda_1}{\cos \lambda_2}
\]

(8.16)

\[
v(\lambda_2) = [v(\lambda_1) + \Omega R \cos \lambda_1] \frac{\cos \lambda_1}{\cos \lambda_2} - \Omega R \cos \lambda_2
\]

(8.17)

The change in zonal velocity (as measured by an observer fixed on Earth) estimated using (8.17) is independent of the magnitude of the meridional velocity; it depends only on the magnitude of the initial zonal velocity and the values of the initial and final latitudes. If the meridional speed is large, the change in zonal speed will occur rapidly; the time required for an air parcel to move from latitude \( \lambda_1 \) to \( \lambda_2 \) is inversely proportional to the magnitude of the meridional wind speed. We wish to interpret the change in magnitude

\[
\begin{array}{c|c|c}
\lambda & v_o(\lambda) & v(\lambda) \\
0 & 4.65 \times 10^4 & 0 \\
15 & 4.81 \times 10^4 & 3.2 \times 10^3 \\
30 & 5.37 \times 10^4 & 1.3 \times 10^4 \\
45 & 6.58 \times 10^4 & 3.3 \times 10^4 \\
60 & 9.32 \times 10^4 & 7.0 \times 10^4 \\
75 & 1.80 \times 10^3 & 1.7 \times 10^3 \\
\end{array}
\]

The change in zonal velocity (as measured by an observer fixed on Earth) estimated using (8.17) is independent of the magnitude of the meridional velocity; it depends only on the magnitude of the initial zonal velocity and the values of the initial and final latitudes. If the meridional speed is large, the change in zonal speed will occur rapidly; the time required for an air parcel to move from latitude \( \lambda_1 \) to \( \lambda_2 \) is inversely proportional to the magnitude of the meridional wind speed. We wish to interpret the change in magnitude
of the zonal wind as the result of a force operating in the zonal direction in the rotating coordinate system. Force, according to Newton's law of motion, is proportional to the rate of change of velocity (see equation 2.1). It follows that the magnitude of the west-east Coriolis force should be proportional to the magnitude of the meridional velocity.

The change in zonal wind directly depends on the speed of planetary rotation. This is most easily seen by considering an air parcel in solid body rotation at \( \lambda_1 \), moving to \( \lambda_2 \). Setting \( v(\lambda_1) \) equal to zero in (8.17), we find

\[
v(\lambda_2) = |0 + \Omega R \cos \lambda_1| \cos \lambda_2 - \Omega R \cos \lambda_2 \quad (8.18)
\]

It follows that the zonal component of the Coriolis force should be proportional to the angular velocity describing the rate of rotation of the planet, \( \Omega \). This completes our qualitative discussion of the zonal component of the Coriolis force. We conclude that the force should be proportional to the meridional velocity and to the value of \( \Omega \); in addition, the magnitude of the force must vanish at the equator. A rigorous analysis indicates that the force per unit mass satisfying these constraints is given by

\[
F_c(w \rightarrow e) = 2 \Omega v_m \sin \lambda, \quad (8.19)
\]

where \( F_c(w \rightarrow e) \) denotes the west-to-east Coriolis force and \( v_m \) denotes the magnitude of the meridional wind speed defined as positive for motion directed toward higher latitude. Recall that the acceleration is force per unit mass. Technically, the right-hand side of equation (8.19) is the Coriolis acceleration, or, equivalently, the force applied to unit mass.

### 8.4 Centrifugal Force and the North-South Component of the Coriolis Force

We turn our attention now to forces influencing motion in the meridional direction on a rotating planet. Consider a particle moving at constant speed, \( v \), on the circumference of a circle of radius, \( R \). For the particle to continue in circular motion, it must experience a force directed toward the center of the circle. This is most easy to visualize by supposing that the particle is attached to a string being whirled around, as discussed in Chapter 2. The force toward the center of the circle is provided by the tension in the string. If this were the only force operating on the particle, we might expect, by Newton's law of motion, that the particle would accelerate toward the center of the circle. The tension in the string is opposed, however, by a force of equal magnitude in the opposite direction to the force exerted by the string. This is the force you feel if you try to ride a bicycle at high speed on a circular track; you have a tendency to fall off the bike to the outside of the track if you fail to compensate by leaning to the inside. The force that tends to drive a particle in circular motion away from the center of its circular path is known as the centrifugal force. (This force was introduced in Chapter 2.) The centrifugal force per unit mass, \( F_{\text{centr}} \), for circular motion is proportional to the square of the absolute speed and inversely proportional to the radius of the circle:

\[
F_{\text{centr}} = \frac{v^2}{R} \quad (8.20)
\]

Consider an air parcel of unit mass moving in the west-east direction at latitude \( \lambda \). The speed of the parcel in the inertial system, \( v_m \), includes a component due to planetary rotation given by (8.4), in addition to a contribution associated with motion relative to Earth, represented by a zonal wind of speed \( v \). The parcel is moving in a circle of radius \( R \cos \lambda \), as indicated in Figure 8.10. The centrifugal force on the parcel is given by

\[
F_{\text{centr}} = \frac{(\Omega R \cos \lambda + v)^2}{R \cos \lambda}
\]

Evaluating \( F_{\text{centr}} \) with the wind speed set equal to \( 10^3 \text{ cm sec}^{-1} \), with \( \lambda \) taken equal to 60°, we find that the three terms in (8.21) have rather different magnitudes; they are equal to 1.7 (term 1, \( \Omega^2 R \cos \lambda \)), 0.15 (term 2, \( 2 \Omega v \)) and \( 3.1 \times 10^{-3} \) (term 3, \( v^2/R \cos \lambda \)), expressed in units of cm sec\(^{-2} \).

Little error is introduced by neglecting the third term. The first term is independent of the magnitude of the zonal wind; it depends simply on latitude and on the angular velocity and radius of the planet. The second term defines the component of the centrifugal force associated with motion relative to Earth.

The first term in (8.21) describes the centrifugal force associated with the rotation of the solid planet and is usually incorporated in the acceleration of gravity by modifying the value of \( g \). The centrifugal force due to rotation of the solid body is responsible for a difference in the equatorial, as compared with the polar, radius of Earth: the equatorial radius is larger than the polar radius by about 21.3 km, as a result of the dependence of the centrifugal force on latitude.

For present purposes we are primarily interested in the force due to motion of an air parcel relative to Earth. The magnitude of this force is given, according to (8.21), by

\[
F_{\text{centr}} = 2 \Omega v \sin \lambda \quad (8.22)
\]

where the superscript \( r \) is included to emphasize that the force in (8.22) refers to motion relative to Earth. The centrifugal force due to zonal motion operates in the direction indicated in Figure 8.13. The component of the force southward is obtained using the cosine rule discussed in Chapter 2: by multiplying the magnitude of the force given in (8.22) by the cosine of the angle separating the southerly direction from the actual orientation of the force, indicated as \( \theta \) in Figure 8.13. The angle \( \theta \) is the complement of the latitude angle \( \lambda \); meaning, \( \theta \) equals \( \pi/2 - \lambda \). Using an elementary trigonometric identity, \( \cos(\pi/2 - \lambda) = \sin \lambda \), the southerly component of the centrifugal force due to west-east relative motion may be expressed in the form

\[
F_{\text{centr}}(n \rightarrow s) = 2 \Omega v \sin \lambda 
\]
Figure 8.13 The direction of the centrifugal force at latitude $\lambda$ for a particle rotating eastward. The component of the force southward, shown by the dashed line, is obtained by multiplying the magnitude of the force by $\cos \theta = \sin \lambda$.

Note that the expression for the north-south component of the centrifugal force due to west-east relative motion is identical to the expression obtained earlier for the zonal component of the Coriolis force, with the meridional speed in (8.19) replaced by the zonal speed in (8.23). The meridional component of the centrifugal force associated with zonal motion is equivalent to the meridional component of the Coriolis force.

Suppose that the zonal motion represents an easterly wind, rather than the westerly wind treated above. The analysis is identical for this case, with $v$ replaced by $-v$; the meridional component of the Coriolis force for an easterly wind operates northward. The Coriolis force, both for meridional and zonal motion, tends to deflect air parcels to the right in the Northern Hemisphere. It is easy to show that it acts to deflect parcels in the Southern Hemisphere to the left, as indicated for zonal motion in Figure 8.14 and as was explicitly demonstrated for meridional motion previously.

The functional form of the Coriolis force is identical for zonal and meridional motion. The force operates in both cases at right angles to the direction of motion: to the right in the Northern Hemisphere, to the left in the Southern. The vector describing horizontal motion in an arbitrary direction at latitude $\lambda$ may be written as a vector sum of the appropriate components for meridional and zonal motion. It follows that the Coriolis force for horizontal motion in an arbitrary direction should operate, as it does for meridional and zonal motion separately, at right angles to the direction of motion—to the right in the Northern Hemisphere, to the left in the Southern—and that the magnitude of the force is given by $2\Omega v \sin \lambda$, where $v$ defines the magnitude of the velocity vector (the wind speed).

$$F = 2\Omega v \sin \lambda \quad (8.24)$$

Expressed in vector notation, the Coriolis force is equal to $2\Omega \times v$.

Figure 8.14 Same as in Figure 8.13 but for the Southern Hemisphere.

8.5 Hadley’s Circulation Revisited

Table 8.2 summarizes values for the zonal wind speed computed as a function of latitude for air moving northward in the upward branch of the Hadley circulation. It was assumed, in constructing the Table, that air was initially at rest with respect to Earth’s equator and that angular momentum was conserved as the air moved northward. The zonal wind speed rapidly increases with latitude, reaching values comparable to the speed of sound at a latitude of about 40°. It would be impossible to maintain a wind speed higher than about a tenth of the sound speed. The Hadley circulation terminates in the present atmosphere at a latitude of about 30°. Air sinks to the surface and the return flow to the tropics takes place near the ground, deflected to the west by the Coriolis force. This accounts for the phenomenon of the trade winds. Hadley was successful in his primary objective to explain "the causes of the General Trade-Winds." His explanation for the midlatitude westerlies is less satisfactory but in no way detracts from the significance of his contributions to meteorology. His achievement is all the more astounding when we recall that Hadley was a contemporary of Newton, that he preceded Coriolis by more than a century, and that his analysis was carried out using qualitative reasoning without benefit of complex mathematics.

8.6 The Concept of Geostrophy

Our discussion to this point has emphasized the role of four basic forces in regulating atmospheric structure and dynamics. The vertical structure of the atmosphere reflects a balance between the vertical pressure gradient pushing air upward and the force of gravity pulling it downward. This balance is expressed through the barometric law, equations (7.14) and (7.15). Motions in the horizontal plane are regulated primarily by the horizontal pressure gradient and by the Coriolis force, with a contribution near the surface from
the force of friction. The wind resulting from a balance between the pressure gradient and Coriolis force is known as the geostrophic wind (from geo meaning "earth" and strophic, meaning "turning"). The geostrophic wind provides an excellent approximation to the real wind at mid to high latitudes (remember that the Coriolis force vanishes at the equator), for motions of moderately large spatial scale (in the absence of sharp gradients in pressure or temperature, or exceptionally high-wind speeds) at altitudes sufficiently far above the surface such that effects of friction may be neglected (a few kilometers or higher). We now discuss how an air parcel, initially at rest, may be expected to move in response to a pressure gradient and how its motion evolves to achieve the state of geostrophic balance.

Consider a parcel of unit volume in the Northern Hemisphere, with a gradient of pressure directed from left to right in a horizontal plane, as indicated in Figure 8.15a. Pressure to the left is higher than pressure to the right. As a result, the parcel is subject to a net force to the right with an associated acceleration, as illustrated in Figure 8.15b. As the parcel begins to move, it experiences the Coriolis force and is turned to the right with respect to its initial direction of motion (i.e., it turns toward the reader, as shown in Figure 8.15c). The parcel rotates until it moves at right angles to its initial direction of motion, as illustrated in Figure 8.15d. The Coriolis force now opposes the force associated with the pressure gradient. The parcel adjusts its speed such that the Coriolis force exactly cancels the force associated with the pressure gradient. The net force on the parcel vanishes, the acceleration drops to zero, and the parcel moves toward the reader at a constant speed determined by the magnitude of the left-right pressure gradient.

Consider a hypothetical spatial variation of pressure, as illustrated for a fixed altitude in Figure 8.16. A parcel of air that is initially stationary will first tend to move in the direction of lower pressure (toward the center of the low) at right angles to the pressure contour on which it is initially located. As it moves, it turns to the right in the Northern Hemisphere; it begins to circle in a counterclockwise direction around the low. In geostrophic balance, motion follows contours of constant pressure. Where contours of constant pressure are relatively far apart, motion is slow (the speed required for the Coriolis force to balance the pressure gradient is low); where contours are closely spaced, speeds are high. The pattern of the pressure field provides an instantaneous snapshot of the field of motion. Air in the Northern Hemisphere moves maintaining high pressure to the right of the direction of motion. The airflow is in a clockwise direction around a low in the Southern Hemisphere, with high pressure to the left. Flow around a low pressure system is said to be cyclonic.

Figure 8.17 illustrates the pattern of flow around a high pressure system. An air parcel that is initially stationary first tends to move away from the high, as indicated. Turning to the right in the Northern Hemisphere under the influence of the Coriolis force, it proceeds to move clockwise around the high, keeping high pressure to the right and following contours of constant pressure in the same fashion as for motion around the low. The pattern is reversed in the Southern Hemisphere; motion around a high is in a counterclockwise direction in this case, with high pressure to the left. The circulation of air around a high is said to be anticyclonic.

Meteorological data are presented most frequently not as maps of pressure at a fixed altitude (as in Figures 8.2 and 8.3) but as contours giving altitudes corresponding to fixed values of pressure. A typical situation is illustrated in Plate 3 (top) (see color insert). Heights of the 500 mb surface for 7 A.M. (EST) on 13 March 1993, varied from a low of 5160 m over northeastern Canada to a high of about 5800 m over the southern
United States. A high value for altitude on this map would correspond to a region of high pressure on a map displaying variations in pressure at fixed altitude, as may be seen in the schematic profiles of pressure versus altitude in Figure 8.18. Suppose that the profile of pressure versus altitude is given for one location by curve 1, while the profile over a neighboring region is summarized by curve 2. A map giving heights corresponding to a pressure of $p_1$ would indicate that the pressure surface was high (altitude $z_1$) over region 1, low ($z_2$) over region 2. At the same time, as anticipated, the map of pressure at altitude $z_2$ would indicate a region of high pressure ($p_1$) over 1, with low pressure ($p_2$) over region 2.

We can think of the contours in Plate 3 (top) as defining a complex, three-dimensional surface. The pressure is constant everywhere on the surface, equal to 500 mb. Altitudes vary in a regular fashion as indicated; there are high regions to the west and east, with a depression in the central portion of the continent. The total relief amounts to about 500 m. Meteorologists, using an obvious geometric analogy, refer to elongated high and low regions on a map defining heights of the pressure field for a given value of pressure as ridges and troughs. As required by the barometric law, temperatures are relatively high in air underlying a ridge, low in the vicinity of a trough. Localized highs, such as the ridges in Plate 3 (top), would be associated with anticyclonic motion, while cyclonic motion would dominate in the vicinity of regional lows or depressions.

Geostrophic wind is directed along contours of constant height on a fixed pressure surface. The wind speed is proportional to the gradient of the height field, just as it is proportional to the pressure gradient on a constant-height map. The more closely bunched the contours in a map (such as presented in Plate 3), the larger the gradient of the pressure-height field and the higher the speed of the geostrophic wind. Wind speeds, expressed in knots (1 knot = 1 nautical mile per hour = 1.85 km per hour), are included in Plate 3, using a compact notation employed by meteorologists as explained in Figure 8.19. Winds on a constant-pressure surface blow in such a direction as to maintain ridges to the right in the Northern Hemisphere and to the left in the south, similar to the orientation with respect to pressure for geostrophic motion on a constant-height surface as discussed above.

It is relatively easy to calculate the speed of the geostrophic wind for a fixed height, given information on the magnitude of the pressure gradient and the mass density $\rho$. Assume that the situation depicted in Figure 8.20 applies to the Northern Hemisphere. The line ABC is drawn as normal (perpendicular) to the pressure contour at B and represents a distance of 100 km. The pressure falls by 4 mb in moving from A to C. Assume that the drop in pressure is linear with distance along AC: the gradient of pressure at B equals $4 \times 10^{-7}$ mb cm$^{-1}$, obtained by dividing the total change in pressure from A to B by the distance from A to B. This defines the magnitude of the pressure gradient $\Delta p/\Delta s$, where $\Delta p$ denotes the change in $p$ taking place over distance $\Delta s$, expressed in units of mb cm$^{-1}$. Converting to cgs units (1 mb equals $10^3$ dyn cm$^{-2}$), the force on a cubic centimeter due to the pressure gradient equals $4 \times 10^{-4}$ dyn cm$^{-3}$.

The Coriolis force on the same cubic centimeter has magnitude $2\rho \Omega v_g \sin \lambda$ (equation 8.24), where $v_g$ is the wind speed and $\lambda$ is latitude as before. Equating the pressure force with the Coriolis force gives

$$\frac{\Delta p}{\Delta s} = 2\rho \Omega v_g \sin \lambda,$$

where the subscript $g$ is included to emphasize that (8.25) reflects the assumption of geostrophy.

Solving for $v_g$, we find

$$v_g = \frac{1}{2\rho \sin \lambda} \left( \frac{\Delta p}{\Delta s} \right).$$

Example 8.5: Find the speed and direction of the geostrophic wind at point B in Figure 8.20, for a latitude of 30° N, with $\rho$ taken as $7 \times 10^{-4}$ g cm$^{-3}$, as appropriate for a pressure of about 500 mb.

Answer:

$$v_g = \frac{4 \times 10^{-4} \text{ dyn cm}^{-3}}{2(7.3 \times 10^{-3} \text{ sec}^{-1})(7 \times 10^{-4} \text{ g cm}^{-3})(5 \times 10^{-1})}$$

$$= 7.8 \times 10^3 \text{ cm sec}^{-1}$$

The speed of the geostrophic wind is 78 m sec$^{-1}$, oriented along the pressure contour at B in the direction indicated by the arrow. If the configuration of the pressure contours in Figure 8.20 were applied to the Southern rather than to the Northern Hemisphere, the wind speed would be the same; the velocity would be reversed, however, and the wind would blow from right to left.

8.7 Effects of Friction

Consider now the effect of friction on the circulation around a high pressure system near the surface, as illustrated in Figure 8.21a. In the absence of friction, air in the
Northern Hemisphere would describe a clockwise path around the high, following a contour of constant pressure. Orientations of the pressure gradient and Coriolis forces for a representative air parcel are indicated in the diagram. Friction leads to a decrease in the magnitude of the wind speed, with an associated reduction in the magnitude of the Coriolis force. Due to the weakening of the Coriolis force by friction, the geostrophic balance is disrupted. This favors the pressure force pushing the parcel in the direction of the gradient. The parcel spirals away from the center of the high, describing the path schematically indicated by the dashed line.

A similar correction occurs for frictional motion around a low, as illustrated in Figure 8.21b. In this case, friction causes the parcel to spiral inward. A typical trajectory for the Northern Hemisphere is indicated by the dashed line. Convergence of air toward the center of the low tends to reduce the magnitude of the pressure gradient, building up mass toward the center of the low and cutting off the flow. This tendency may be offset, however,

**Figure 8.18** Profiles of pressure versus altitude for two different positions, labeled (1) and (2). The heights of the surface corresponding to a pressure of $p_1$ is $z_1$ at (1) and $z_2$ at (2). Pressure at altitude $z_1$ is higher at (1) where it is equal to $p_1$, than it is at (2), where it is equal to $p_2$.

<table>
<thead>
<tr>
<th>Simplified surface-station model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature ($^\circ$F)</td>
</tr>
<tr>
<td>Wind direction (from the NW)</td>
</tr>
<tr>
<td>Wind speed (18–22 knots)</td>
</tr>
<tr>
<td>Total amount of clouds (overcast)</td>
</tr>
<tr>
<td>Barometric pressure at sea level (1013.8 mb)</td>
</tr>
<tr>
<td>Pressure tendency during past 3 hours (rising)</td>
</tr>
<tr>
<td>Amount of pressure change in past 3 hours (3.2 mb)</td>
</tr>
<tr>
<td>Sign indicating whether pressure is higher or lower than 3 hours ago</td>
</tr>
<tr>
<td>Present weather (light rain)</td>
</tr>
<tr>
<td>Dew point ($^\circ$F)</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

**Figure 8.19** Notation employed by meteorologists on their maps. Source: Ahrens 1994.
Fig. 8.20  Schematic map of pressure surfaces for an altitude of about 6 km. It is assumed that the x and y axes measure distances to the east and north, respectively.

Fig. 8.21a  Schematic illustration of the trajectory of an air parcel at the surface in the Northern Hemisphere in the vicinity of a high pressure system, showing the effect of friction. Direction of Coriolis force indicated by arrows on trajectory.

Fig. 8.21b  Similar to 8.21a, illustrating the trajectory of an air parcel at the surface in the Northern Hemisphere in the vicinity of a low pressure system. Direction of Coriolis force indicated by arrows on trajectory.

by passage of a low pressure system aloft. In this case, air drawn to the low will be forced to rise, resulting in unsettled weather. If the low pressure system aloft is well developed, and if the moisture content of the surface air is relatively high, the rising motion can result in intense precipitation. Hurricanes provide an extreme example of this phenomenon. The change in pressure associated with Hurricane Bob, a violent storm that caused major damage to coastal areas of New England in August 1991, is illustrated in Fig. 8.22. Conversely, passage of a high pres-

sure system aloft will cause the atmosphere to descend, with an outflow of surface air. Descending air is normally dry. Temperatures will rise in this case as a result of adiabatic compression. High pressure systems are usually associated with warm weather and clear skies, especially during summer.

8.8 Further Perspectives on the General Circulation

As we have seen, Hadley’s model, modified to account for conservation of angular momentum rather than absolute speed, provides a reasonable representation of the general circulation of the atmosphere in the tropics, for latitudes less than 20°–30°. It accounts for the trade winds but fails to provide an explanation for the prevailing surface westerlies at midlatitudes. Figure 8.23 presents an alternate model proposed more than a century later, in 1856, by an English schoolteacher named William Ferrel.

Ferrell’s model envisages a circulation with three distinct loops or cells in each hemisphere. The picture in Fig. 8.23 indicates a circulation system in the tropics similar to the model proposed by Hadley for the globe. The tropical cells extend to the region of the Horse Latitudes, at about 30°. The cells at intermediate latitudes, 30°–60°, however, have air revolving in a sense opposite to the flow in the tropics. Air rises on the poleward branch of the intermediate cell. It flows southerly in the upward branch in the Northern Hemisphere, returning to the surface at the southern margin. The loop is completed by northward transport near the ground. The high latitude (60°–90°) cells are characterized by sinking motion near the

Fig. 8.22  Surface pressure vs. time recorded at Ipswich, Massachusetts, during the passage of Hurricane Bob. Data courtesy of Quinn Sloan.
poles, with circulation in the same sense as for the tropics. Sharp thermal gradients mark the boundaries between the high temperatures of the tropics, the moderate temperatures at mid-latitudes, and the cold temperatures near the poles. The boundary between the intermediate and polar cells is known as the polar front. The boundary between the tropical and midlatitude cells is associated with the midlatitude jet stream.

Ferrel's model was based on an attempt to provide a rational for three-dimensional features of the general circulation observed at the surface. His model accounts, for example, for the prevailing surface westerlies at mid-latitudes. These arise as a consequence of eastward deflection of winds in the lower branch of the intermediate cells. There are problems, however, with the model, particularly with the intermediate cell. The surface westerlies exert an eastward torque on the solid planet (see Section 2.10). This must be accompanied by a transfer of angular momentum from the atmosphere to the surface. A loss of angular momentum by the atmosphere at mid-latitudes must be balanced by a supply from other regions, most likely from the tropics. But transport of angular momentum in the intermediate cell is from high to low latitudes, rather than in the reverse direction, as required to balance the sink at the surface. Ferrel's model predicts that the winds aloft at mid-latitudes should blow easterly due to effects of the Coriolis force on the equatorward flow. Observations, summarized in Figures 8.24 and 8.25, indicate, however, that westerlies are dominant everywhere except for a limited region in the tropics. There is also a problem with

Figure 8.23 Model of the general circulation proposed by Ferrel (1856), illustrating directions of prevailing winds at the surface, with an exaggerated perspective on the vertical components of the motion field.

Figure 8.24 Three cross-sectional views of zonal (east-west) winds, showing zonal mean wind velocities (i.e., average wind velocity at each latitude). Wind velocities are given in m/sec⁻¹, showing only the zonal components of the wind speed. Positive values imply that the wind is from the west, negative values imply that wind is from the east. Note that altitude is expressed in terms of pressure. One decibar (db) equals 100 millibars (mb). The top panel shows mean annual values, the middle panel shows mean values for December-January-February (DJF), and the bottom panel shows mean values for June-July-August (JJA). Source: Peixoto and Oort 1992.
the transport of heat. Rising motion at high latitudes, accompanied by sinking of air at low latitude, would imply a net transport of heat by the intermediate cell from high to low latitude. This is clearly inconsistent with the current understanding of the atmospheric radiative budget, as summarized in Chapter 7. The difficulty is ultimately linked to limitations inherent in the models presented to date, limitations associated primarily with their assumption that the circulation can be represented in terms of a zonal average. Important transports of angular momentum, and of heat and other dynamic variables, can arise due to departures from zonal symmetry as a consequence of longitudinally dependent features of the circulation. We refer to longitudinally variable characteristics of the circulation as eddies.

8.9 Transport by Eddies

Suppose that the time-averaged value of the meridional velocity at latitude \( \lambda \) and longitude \( \theta \) is given by \( v(\lambda, \theta) \). We write \( v(\lambda, \theta) \) as a sum of two terms: one \( \bar{v}(\lambda) \) defining the average of \( v(\lambda, \theta) \) with respect to time and longitude, the other \( v'(\lambda, \theta) \) providing a correction to account for the time-averaged value of \( v(\lambda, \theta) \) at \( \theta \):

\[
v(\lambda, \theta) = \bar{v}(\lambda) + v'(\lambda, \theta)
\]  

Assume, for the moment, that the averages over time in (8.28) refer to means with respect to a season. The quantity \( \bar{v}(\lambda) \) is the meridional analogue to the time- and longitudinally averaged zonal velocity presented in Figures 8.24 and 8.25. By definition, \( \bar{v}(\lambda) \) specifies the average speed with which air is transported across latitude over the course of a season; it is the seasonally averaged meridional speed at latitude \( \lambda \). On the other hand, \( v' (\lambda, \theta) \) represents the time averaged variance of \( v(\lambda, \theta) \) with respect to the zonally (longitudinally) averaged quantity \( \bar{v} \).

Consider the simple variation of \( v(\lambda, \theta) \) with \( \theta \) indicated in Figure 8.26a. The time-averaged speed of the meridional wind is set equal to 2 m sec\(^{-1}\), except in the regions denoted by A and B. The northward speed is assumed equal to 3 m sec\(^{-1}\) in region A. The wind is taken to blow southward in region B at a speed of 1 m sec\(^{-1}\). We suppose that region A is three times more extensive than region B. It follows that the speed of the average wind associated with the combination of regions A and B is identical to the average for the meridional wind at other longitudes, 2 m sec\(^{-1}\) to the north.

Corresponding values of \( v' (\lambda, \theta) \) are given in Figure 8.26b. We identify \( v' \) as the velocity associated with eddies, or departures from the zonal mean of 2 m sec\(^{-1}\). The contribution of eddies to the transport of air across a latitude circle is exactly zero, by definition. Excess transport northward in
Standing eddies are often associated with specific geographic features. They can arise, for example, as a result of thermal contrasts between land and sea as a function of longitude at a particular latitude. The Indian Monsoon, induced by intense heating of the Indian subcontinent in summer and by inflow of moist air from the surrounding ocean, provides a useful example of a large-scale standing eddy. In addition to standing eddies, there are also fluctuations in velocity that are more ephemeral. We refer to these more rapid variations as transient eddies. They are associated, for example, with the passage of weather systems, high and low pressure systems moving rapidly through particular regions. Seasonally averaged maps show little trace of transient eddies; northward fluctuations in velocity occur just as often as southward fluctuations at particular locations, and net contributions from northward and southward motions tend to cancel when data are averaged over a period as long as a season. Transient eddies, nonetheless, can account for significant transport of dynamical variables such as heat and angular momentum and must be recognized as playing an important role in the general circulation of the atmosphere.

Contributions of transient eddies, stationary eddies, and mean circulation to meridional transport of sensible heat are summarized in Figure 8.27. Note the dominant role of transport by eddies in both hemispheres for both winter and summer at latitudes higher than about 30° (Figure 8.27, panels b and c). Transport by the mean circulation plays an important role in the tropics (Figure 8.27d). Transport by the mean circulation is directed southward over the latitude band 30°N to about 18°N during Northern Hemispheric winter. The direction of transport by the mean circulation switches northward over the latitude band 30°N to about 18°N during Northern Hemispheric summer. The large seasonal swing in the direction of transport by the mean circulation in the tropics as indicated in Figure 8.27d reflects the influence of the Indian, or South Asian, monsoon. The monsoonal flow is northeasterly during Northern Hemispheric winter, southwesterly during Northern Hemispheric summer. Averaged over the course of a year, transport of sensible heat in the tropics is directed equatorward from the hot desert regions characteristic of the descending loops of the Hadley circulation. Accounting for all forms of energy (sensible heat, latent heat, and potential energy), net transport of energy over the course of a year is directed away from the equator in both hemispheres.

8.10 The Angular Momentum Budget of the Atmosphere

A discussion of the angular momentum budget of the atmosphere is necessary for a refined understanding of atmospheric circulation. Hadley’s and Ferrel’s models were deficient because of their failure to account for a balanced angular momentum budget. Taking the conservation of angular momentum into account is the key to comprehending why westerlies predominate in mid- to high latitudes. Toward this end, we must answer the following questions: what are the sources and sinks of atmospheric angular momentum, how is angular momentum transferred from sources to sinks, and how is this transfer of angular momentum associated with the observed wind patterns?
reduce the speed of the surface wind, transferring linear momentum (see Chapter 2) from the atmosphere to the liquid or the solid planet with which it is in contact. Frictional loss of linear momentum by the atmosphere is accompanied by a corresponding gain of the same by the rest of the planet. A change in linear momentum, by Newton’s law of motion, implies a force. The force of friction on the air opposes the direction of the wind (friction causes a decrease in the speed and consequently a decrease in the linear momentum of the air measured with respect to its direction of motion). The corresponding force on Earth is in the direction of the wind.

Defining angular momentum in the direction of planetary rotation (eastward) as positive, it follows that an eastward force implies a positive torque and acts to increase angular momentum. A westward force implies a negative torque and acts to decrease angular momentum. If the wind is westward (i.e., easterly), the force of friction on the air is eastward; the atmosphere gains angular momentum at the expense of the solid-liquid planet. An easterly wind near the surface exerts a westward frictional torque on the planet: the angular momentum of the planet’s solid and liquid region decreases as a consequence, resulting in a decrease in the rate of rotation, with a corresponding increase in the length of the day. Fortunately, this is balanced elsewhere by westerly winds that are responsible for a reduction in the angular momentum of the atmosphere, with a compensatory increase in the planet’s angular momentum. The angular momentum of Earth as a whole—atmosphere plus solid and liquid components—is conserved. Angular momentum is communicated to the atmosphere from the underlying planet in one region, removed in another. Sources and sinks of angular momentum must balance separately over time on a global scale for the atmosphere, ocean, and solid planet. An imbalance, maintained for any appreciable time, would result in a detectable change in the length of the day.3

Figure 8.27 The northward flux of sensible heat, averaged over all latitudes and altitudes. The horizontal axis shows latitudes from 80°S to 80°N, and the vertical shows the heat flux in units of m sec⁻¹ °C. The four panels show transport by (a) all motion, (b) transient eddies, (c) stationary eddies, and (d) mean meridional circulation. Dashed lines refer to months June-August; light solid lines are for December-February; solid lines define annual averages. Source: Peixoto and Oort 1992.

Exchange of angular momentum between the atmosphere and surface is accomplished primarily by torques imposed by the force of friction, with an additional component due to torques associated with west-east gradients in atmospheric pressure operating on north-south-aligned mountain chains such as the Rockies and the Andes. Friction acts to

Figure 8.28 The zonal-mean (average at each latitude) stress imposed on the atmosphere by land plus ocean, expressed in dyn cm⁻², for the whole year (panel a), December through February (DJF) (panel b), and June through August (JJA) (panel c). Source: Peixoto and Oort 1992.
terized by a cell-like movement of an air parcel, meaning that surface winds travel in a direction opposite to the winds aloft. Surface winds may blow southwestward while winds aloft may blow northeastward. At upper latitudes, the surface winds blow largely in the same zonal direction as the winds aloft, although at different speeds.

Estimates for the net northward transport of zonal momentum by the atmosphere (expressed as the product of the northward and eastward wind speeds) are presented in Figure 8.30. Net transport by the composite of all motions is summarized in panel (a). Transport by transient eddies, standing eddies, and mean meridional motion is indicated in panels (b)–(d).

Note the importance of transport by eddies for all latitudes. One can think of angular momentum as accumulating in the atmosphere in the tropics in the region of the easterly trade winds. It is expended at higher latitudes in the region of the surface westerlies. The bulk of the latitudinal redistribution of angular momentum by the atmosphere takes place in the upper troposphere, as indicated in Figure 8.31. The largest contribution originates from the vicinity of the midlatitude jets, from the region where the zonal wind is a maximum near the high-latitude edges of the Hadley regime (see Figures 8.24 and 8.25). Transport of angular momentum differs from transport of heat in that angular momentum is exchanged more readily between different air parcels.

The picture of the circulation presented by Ferrel is conceptually useful but in some respects misleading. The circulation at mid- and high latitudes is governed by complex eddies rather than by an organized cell (or cells) as suggested in Figure 8.23. It is important to distinguish between the role of the mean (Hadley) circulation at low latitudes and the more complex effects of eddies at higher latitudes. The low latitudes are charac-

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**Figure 8.29** The torque exerted on the atmosphere by oceans and land at different latitudes (left-hand panel). The right-hand panel gives the torque exerted by mountains. Results are integrated over 5°C latitude bands and expressed in units of $10^{18}$ kg m² sec⁻². Separate curves refer to summer (June-August, JJ), winter (December-February, DJ), and annual mean.

**Figure 8.30** Northward transport of angular momentum averaged over latitude and altitude. Note that southward transport is represented by negative values. Panel (a) shows transport by all motion, panel (b) transport by transient eddies, panel (c) transport by stationary eddies, and panel (d) transport by mean meridional circulation. Data are presented in units of m² sec⁻². Seasonal and annual means are indicated as in Figures 8.28 and 8.30. Source: Peixoto and Oort 1992.
The Angular Momentum Budget of the Atmosphere

Figure 8.31 Annual average zonal-mean cross sections of the northward flux of angular momentum by (a) all motions, (b) transient eddies, (c) stationary eddies, and (d) mean meridional circulation. Units are m$^2$ sec$^{-2}$.

The question remains as to the physical mechanism by which surplus angular momentum is transferred to higher latitudes. Exchange of angular momentum between different atmospheric regions is accomplished primarily through torques that contort the zonal flow of air. In the Northern Hemisphere, the torques add angular momentum (an eastward force) to northward moving parcels and remove angular momentum (a westward force) from southward moving parcels. These torques are associated with zonal gradients of pressure. Thus, pressure gradients on a given latitude are ultimately linked to the existence of eddies.

The meridional transfer of angular momentum must be achieved through a wavelike pattern such that mass is conserved. Not every wavelike pattern will suffice, however, as some patterns are more efficient than others in transferring angular momentum. For example, the sinusoidal wave pattern in Figure 8.32 is not efficient in transporting angular momentum.

Example 8.6: Calculate the net angular momentum transferred from 30$^\circ$N to 35$^\circ$N by a 1 g air parcel traveling in one sinusoidal wavelength (refer to Figure 8.32). The west-to-east component of velocity for points A, B, and C are all 30 m sec$^{-1}$.

Answer: The net angular momentum transferred is zero. The angular momentum transferred northward from point A to point B exactly equals the angular momentum transferred southward from point B to point C.

A more typical configuration of transient high and low pressure systems is illustrated in Figure 8.33. The streamlines of the flow depict the instantaneous directions of the

Figure 8.32 Sinusoidal fluctuations of zonal winds with associated highs and lows. This type of flow is inefficient in transporting angular momentum.
wind velocity. Note that the flow follows a clockwise pattern around the high and a counterclockwise pattern around the low, as expected for geostrophic balance. Furthermore, note that the air moving northward has a strong westerly component: its angular momentum, defined by the magnitude of the wind velocity eastward, is relatively high. The air moving southward has a relatively low angular momentum: the westerly component of the wind velocity in this case is small.

![Diagram of zonal motion and angular momentum](image)

**Figure 8.33** A typical fluctuation in the zonal motion illustrating the role of eddies in transporting angular momentum. In the Northern Hemisphere, note the strong easterly component of the flow northward as compared to the weak component of zonal motion associated with meanders of flow southward. Source: Peixoto and Oort 1992.

**Example 8.7:** Calculate the net angular momentum transferred from 30° to 35° by a 1 g air parcel (refer to Figure 8.33). Assume west-east velocities differ at points X, Y, and Z as follows:

\[
V_X(30^\circ) = V_Z(30^\circ) = 30 \text{ m sec}^{-1}
\]
\[
V_Y(35^\circ) = 0
\]

Further, assume that the wind between points Y and Z has no zonal component, meaning it blows straight southward.

**Answer:** Recall that angular momentum is defined as

\[
L(\lambda) = V(\lambda)R \cos(\lambda)
\]

\[
L_X(30^\circ) = V(30^\circ)R \cos(30^\circ) = (30 \text{ m sec}^{-1})(6.378 \times 10^6 \text{ m})(0.5) = 9.6 \times 10^7 \text{ m}^2 \text{ sec}^{-1}
\]

\[
L_Y(35^\circ) = V(35^\circ)R \cos(35^\circ) = 0
\]

Note that the angular momentum is transferred northward, while none is transferred southward. This type of eddy is extremely efficient in transferring angular momentum. Transport of angular momentum from low to high latitude by eddies yields strong westerly flows and weak easterly flows, accounting for the predominance of westerly winds in the middle troposphere at nonequatorial latitudes.

The meandering stream lines in Figure 8.33 are associated thus with significant northward transport of angular momentum. This is the case even though the flow pattern in Figure 8.33 is associated with little, or no, latitudinal transfer of mass. Southward transport of mass at Y is almost exactly balanced by northward transport at X.

Northward-moving eddies experience on average a westward pressure gradient while southward eddies are subject on average to an eastward pressure gradient. Thus, air parcels moving north at X experience a negative torque associated with the component of the pressure force operating westward. They lose angular momentum as they move. The angular momentum lost in this fashion is communicated to the air through which the parcels move. The surrounding air, then, has an increased flow from west to east. Similarly, parcels moving southward at Y are subjected to an eastward pressure force. Angular momentum is extracted from the ambient air in this case as a result of the positive west-east torque experienced by the southward-moving parcels. The angular-momentum surplus acquired on the southward excursion is returned to the ambient atmosphere on the flow's subsequent northward meander.

A similar mechanism operates in the Southern Hemisphere. Transport of angular momentum from low to high latitudes by eddies in both hemispheres plays a dominant role in the global redistribution of angular momentum. It provides the source required to balance the atmosphere's loss of angular momentum to the surface at midlatitudes associated with the negative torque exerted on the atmosphere by the planet in conjunction with the prevailing surface westerlies in this region. Westerly winds are ultimately required at midlatitudes to balance the negative torque exerted on Earth by the surface easterly trade winds at low latitudes. In this sense, eddies are an essential component of the global circulation of the atmosphere. They play a dominant role in the heat and angular momentum budget of the atmosphere at all latitudes poleward of about 20°.

### 8.11 The Origin of Eddies

Horizontal motions of the atmosphere are predominantly in the zonal (east-west) direction. Eddies typically arise as a result of instabilities in the zonal flow. A kink, a meridional excursion, develops and proceeds to propagate as a wave, similar to the pattern displayed in Figure 8.33. Instabilities are commonly linked to the north-south variation of pressure surfaces that typically slope downward from low to high latitudes. Adjustment of the slope of pressure surfaces can result in the release of significant amounts of energy. This provides the fuel for the growth of eddies. Their subsequent evolution is strongly influenced by rotation and constraints imposed by the need to conserve angular momentum. Densities can significantly vary along surfaces of constant pressure, due to variations in temperature. Eddies feeding on variations of density on constant-pressure surfaces are known as baroclinic eddies. Much of the transient atmospheric motion is associated with baroclinic eddies and is ultimately related to the tendency of the atmosphere to adjust to minimize the density variation on constant-pressure surfaces.
8.12 Summary

Atmospheric structure and temperature are controlled by absorption of solar radiation, mainly in the visible portion of the spectrum, by redistribution of energy absorbed from the Sun through the complex system of winds and eddies associated with the general circulation, and by emission of radiant energy into space, primarily in the infrared region of the spectrum. Solar radiation is most intensely absorbed at low latitudes. Emission of infrared radiation into space is more evenly distributed, such that the net radiative budget of the atmosphere is positive at low latitudes (more energy is absorbed from the Sun than is emitted in the infrared), negative at high latitudes. The radiative surplus at low latitudes is balanced by transport through the general circulation. Sufficient energy is transported by atmospheric and oceanic motions from low to mid and high latitudes of the atmosphere, with an important additional contribution, as will be discussed later, from the ocean to offset the radiative deficit in these regions.

Three distinct dynamical regimes can be identified. The thermal circulation described by Hadley plays an important role in the tropics. Air rises in the region of maximum heating, moving to higher latitudes aloft under the influence of a thermally maintained gradient in pressure. As it moves to higher latitude, air drifts eastward under the influence of the Coriolis force, seeking to conserve angular momentum. Instabilities in a primarily zonal flow give rise to eddies, assisting in the latitudinal transfer of angular momentum and heat, extending the tropical regime to a latitude of about 30° under present climate conditions. In the absence of eddies, the tropical regime would be confined to latitudes below about 20° and tropical temperatures would be most likely higher than today, with temperatures at mid- and high latitudes correspondingly colder.

The high-latitude boundary of the tropical, or Hadley, regime is marked by a strong westerly jet in the middle troposphere. The speed of the jet depends on the meridional pressure gradient, as required by geostrophy. The meridional pressure gradient is in turn linked to the meridional (south-north) gradient in temperature. The more efficient the transport of heat and angular momentum by eddies, the more extensive the tropical regime and the smaller the contrast in temperature between low and high latitudes. Ice ages may correspond to times when the tropical regime was relatively confined. Equable climates (occasions when Earth was uncommonly warm) may represent a response to unusually efficient transport by eddies, with an associated expansion of the tropical regime.

The upper-latitude boundary of the tropical, or Hadley, regime is characterized by the sinking motion of the atmosphere. Since water is removed by precipitation as air rises and cools in the ascending branch of the Hadley cell at the equator, the atmosphere in the descending branch is unusually dry. Temperatures at the surface are high as a result of adiabatic compression. This is the region occupied by the great desert areas on land and the location of the Horse Latitudes so eloquently described by Coleridge for the ocean. Air returns to the equator through the lower branch of the Hadley cell. As it moves southward (northward in the Southern Hemisphere), it is deflected westward by the Coriolis force, accounting for the existence of the trade winds.

Transport of heat and angular momentum is mainly effected by eddies at midlatitudes, particularly during winter. Eddies are manifest at the surface by passage of the high- and low pressure systems that play a dominant role in midlatitude weather. Geostrophic balance implies that the flow of air around a low pressure system, or cyclonic motion, should proceed in a counterclockwise direction in the Northern Hemisphere (high pressure is maintained to the right of the direction of motion). The sense of the flow is reversed for the Southern Hemisphere. Friction leads to a reduction in wind speeds near the surface. This causes air to converge toward the center of the low, spiralling inward and rising as it crosses isobaric (constant pressure) surfaces. Low pressure systems are associated frequently with unsettled weather, with precipitation often the consequence of the cooling of moisture-laden air as it rises. Conversely, high pressure systems are the harbinger of clear, cloud-free conditions, with descending motion dominant at the center of highs, accommodating frictionally driven divergence (or outward flow) of surface winds.

Cyclones, anticyclones, and frequent change are dominant characteristics of the weather at midlatitudes. Angular momentum transported by the atmosphere to high latitudes is dissipated by friction associated with prevailing westerly winds at the surface, in combination with torques relating to pressure gradients operating on north-south–aligned mountain chains. Surface westerlies at midlatitudes are an essential feature of the general circulation; they are required to balance oppositely directed torques imposed on the surface by the easterly trade winds at low latitudes. The indirect cell proposed by Ferrel for the mean circulation at midlatitudes is an idealization at best. It is difficult, indeed impossible, to divine a pattern as simple as the Ferrel cell in a realistic map of the weather. It exists, if at all, as a long-term average of the circulation, after dominant contributions from eddies have been eliminated. Mean circulation plays a trivial role in the transport of heat and angular momentum at all latitudes, except in the tropics. The meteorology of mid- and high latitudes is dominated by eddies.

The transition from midlatitudes to the polar region is marked, especially during winter, by a sharp temperature gradient, an associated pressure gradient, and a strong, geostrophically maintained, largely zonal wind reaching maximum speeds in excess of 100 mph (sometimes as high as 200 mph) at an altitude of about 10 km above the surface. The jet stream provides important assistance to aircraft flying eastward, reducing scheduled travel times from North America to Europe by as much as an hour with respect to the westward return journey. The jet stream undergoes frequent meanders, resulting in rapid surface temperature changes in regions near the margin of the polar environment. It is not uncommon for the jet stream to stick in a particular configuration for periods as long as a week
or more. Regions on the poleward side of the jet stream are then subjected to an unrelenting cold snap, while areas on the equatorward side enjoy prolonged balmy temperatures. Such a condition occurred over the United States between the 15–20 April 1976, as illustrated for 17 April in Plate 3 (bottom) (see color insert). The western portion of the country was unseasonably cold as the southward meander of the jet brought frigid air from Northern Canada as far south as Northern Mexico. South of the jet stream, warm air invading from the sub tropics resulted in record-high temperatures over much of the central and eastern United States. Temperatures rose as high as 88°F in Chicago, while records were broken in Boston, as temperatures climbed into the high 90s. Temperatures dropped by as much as 50°F over a 24-hour period a few days later, as tropical air derived from south of the jet stream was replaced by polar air from the north. Average temperatures for winter in cities such as Boston can vary significantly from year to year under the influence of apparently random migrations of the jet stream. Instabilities, or eddies, forming on the edge of the jet, have an important influence on weather and precipitation for most regions of the world at mid to high latitudes in winter.

Weather on the high-latitude side of the polar front is unrelentingly cold in winter. As is the case at midlatitudes, eddies play a dominant role. Even the concept of a mean circulation is problematic: large interannual variability is the rule rather than the exception, especially for the Northern Hemisphere. The picture of the general circulation displayed for high latitudes in Figure 8.23 is even more idealized than the view presented for midlatitudes. An appreciation of climate at high latitudes, as at midlatitudes, demands an understanding of eddies. Eddies represent the signal, not simply the noise, of the climate system for much of the world.