# GEOL5690: Fold and Thrust Belts and Orogenic Wedges

One of the earlier mysteries in geology was the discovery in the 19th century of large scale overthrusts in the Alps. Sheets of rock were found to have pushed over large regions with the boundary fault being at fairly gentle dips. Windows through the upper plates of these thrusts (fensters, from the German for window) and isolated outliers (klippe, also from German for cliff) confirmed the gentle dip extended well back from the thrust front. Similar thrust sheets characterize the Sevier orogen in the western U.S.

Although geologists accepted these large displacements from fairly early on, geophysicists and rock physicists were skeptical because studies of the strength of rocks suggested that rocks simply were not strong enough to permit such faults to be active. The logic was simple. Imagine a thrust sheet of thickness $h$ and width $w$ made of rock of density $\rho$; the weight per unit area on the base of the sheet is $\rho g h$. If the thrust is flat, then this is the normal stress on the thrust. The force opposing the thrusting is friction; if the coefficient of friction on the basal thrust is $\mu_b$, then the force opposing thrusting per unit length of the thrust is $\mu_b \rho g h w$ (we have dropped a fault strength term here for now).

Now if the force being applied is at maximum, we presumably can drive the largest possible thrust sheet with rocks at failure. One commonly assumed criterion is the Coulomb failure criterion, which dates to the 18th century. This is based on the idea that a fault plane will slip when the shear stress $\tau$ exceeds some intrinsic strength $S_0$ and a frictional coefficient $\mu$ times the normal stress on the fault $\sigma$:

$$|\tau| = S_0 + \mu\sigma \qquad (1)$$

If all possible planes in a rock have the same parameters, then for given principal stresses $\sigma_1$ and $\sigma_3$, we find

$$\begin{aligned} \sigma_1 &= C_0 + q\sigma_3, \text{ where} \\ q &= \left[ \left( \mu^2 + 1 \right)^{1/2} + \mu \right]^2 \\ C_0 &= 2 S_0 q^{1/2} \end{aligned} \qquad (2)$$

To get the maximum force, $\sigma_1$ will be horizontal and $\sigma_3$ will be vertical and equal to $\rho g z$. Then it is simple to show that if we integrate $\sigma_1$ over the edge of the block and balance it against the frictional force

$$\mu_b \rho g h w = C_0 h + \tfrac{1}{2} q \rho g h^2 \qquad (3)$$

Because rocks really don't have a huge intrinsic strength at these scales, $C_0$ is close enough to zero to ignore and we get

$$w = \frac{qh}{2\mu_b} \qquad (4)$$

For values of $\mu_b$ of about 0.6, as is generally observed for crustal rocks, $w$ is about 2-3 times the thickness of the thrust sheet. Of course, if the coefficient of friction were very low, this is ok, but the rocks in the thrusts didn't seem that weak. This was not enough to satisfy the geologists (and we have ignored another problem of the rotational moment of our stresses on the side…). Also, if we have to move the slab uphill, the force will be greater yet and the maximum slab length will decrease.

**Introducing fluid pressure.** The first solution came from Hubbert and Rubey in a series of GSA Bulletin articles near 1959 (this is summarized in section 17.4 of Jaeger and Cook, *Fundamentals of Rock Mechanics*, 3rd Edition). They noted that fluid pressures in rocks reduce frictional resistance. Normal stresses in the Coulomb failure criterion decrease by the fluid pressure:

$$\sigma_1 - \lambda\rho g z = C_0 + q(\sigma_3 - \lambda\rho g z) \quad (5)$$

where $\lambda$ is the ratio of fluid pressure to lithostatic pressure ($\lambda = 0$ is what we have already considered). As we are using the lithostatic pressure for $\sigma_3$, it follows that

$$\sigma_1 = C_0 + \rho g z[q + \lambda(1-q)] \quad (6)$$

The frictional resistance will also decrease to $\mu_b \rho g h(1-\lambda_b)$, where $\lambda_b$ is the fluid pressure ratio at the thrust fault. Repeating the steps into (3) and (4) but with our fluid pressure terms we get:

$$w = \frac{h[q + \lambda(1-q)]}{2\mu_b(1-\lambda_b)} \quad (7)$$

This result will increase the potential width by about an order of magnitude, especially if $\lambda_b$ is larger than $\lambda$.

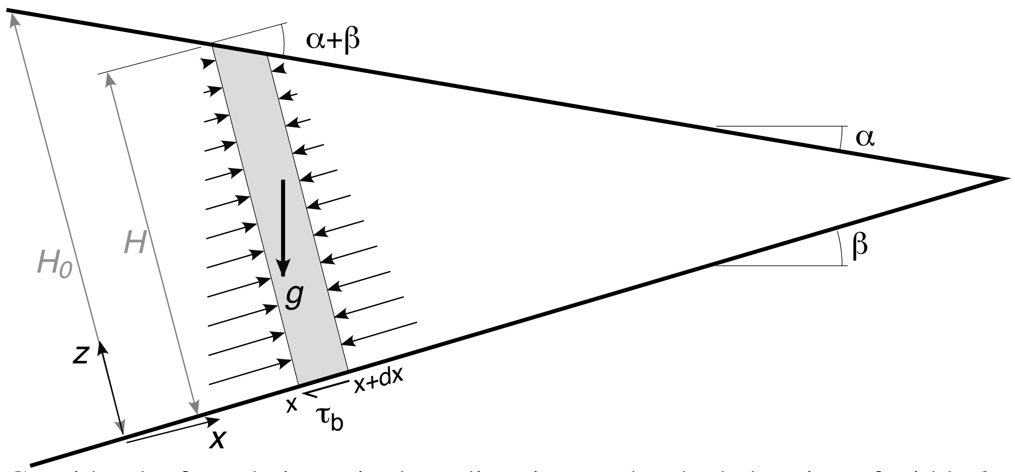
This was (and is) widely regarded as a very clever solution to a nagging problem, and indeed there are numerous measures of pore pressure that indicate that this is a critical issue in thrust faulting. But the solution doesn't get you far with dipping thrusts and hasn't led to too many reinterpretations of geologic histories or new insights into those histories.

Some intermediate work we will ignore focussed on using gravity as the driving force all by itself. This reached apogee in the early 1970s with a volume on Gravity and Tectonics, which carried the seeds for the destruction of this conceptualization. Basically, the hinterland of the orogenic belt had to be prohibitively high to drive forward the thrust sheets with reasonable numbers.

**A wedge-shaped orogen.** The next big step came in the late 1970s. Bill Chapple at Brown recognized that the shape of thrusting orogens was a wedge, not a block, and that a wedge had certain nice self-similarities (however you scale the wedge, the physics stays the same). He focused on plastic rheologies (GSA Bulletin paper in 1979), but it wasn't much farther to using the same Coulomb criterion for brittle failure to understand how you would get a wedge. This was presented by Davis, Suppe, and Dahlen in a paper in JGR in 1983.

Consider a wedge of material; for simplicity we will make it cohesionless ($C_0 = S_0 = 0$) and let it be at failure throughout (we shall return to this later, but it turns out that if the wedge is not at failure, it will deform internally until it recovers its "critical" shape). This is a so-called "Coulomb wedge" or "critical tapered wedge." Fundamentally, the physical solution is to simply increase the cross-sectional area of the wedge as you move away from the thrust. Our simple analysis will reveal how this plays out.

Let us now use stresses in coordinates parallel ($x$) and perpendicular ($z$) to the basal thrust. Then if we look at a small width of wedge we can see how forces will be transmitted. We will assume our wedge is subaerial for simplicity.




Consider the force balance in the $x$ direction on the shaded region of width $dx$. Note that gravity will have a component of force in the negative $x$ direction. We find when we balance this that

$$\tau_b + \rho g H \sin\beta + \frac{d}{dx}\int_0^H \sigma_x dz = 0 \tag{8}$$

If we again assume that the frictional resistance is about equal to the weight times the coefficient of friction $\mu_b$, then

$$\tau_b \cong \mu_b \rho g H \cos\beta(1-\lambda_b) \tag{9}$$

Let us also approximate the normal stress parallel to $x$ as $\sigma_1$ from the rectangular case above (eqn. 6), adjusting for the angle of $z$ from the vertical:

$$\begin{aligned} \frac{d}{dx}\int_0^H \sigma_x dz &= \frac{d}{dx}\int_0^H (q+\lambda(1-q))\rho g z \cos\beta dz \\ &= \frac{d}{dx}\left[\tfrac{1}{2}(q+\lambda(1-q))\rho g \cos\beta H^2\right] \end{aligned} \tag{10}$$

Using simple geometry of $H = H_0 - x\tan(\alpha+\beta)$, we can differentiate this last term to

$$\begin{aligned} \frac{d}{dx}\int_0^H \sigma_x dz &= \frac{d}{dx}\left[\tfrac{1}{2}(q+\lambda(1-q))\rho g \cos\beta\left(H_0 - x\tan(\alpha+\beta)\right)^2\right] \\ &= (q+\lambda(1-q))\rho g H \tan(\alpha+\beta) \end{aligned} \tag{11}$$

Combining (11) and (9) into (8) and using small angle approximations, we find

$$\begin{aligned} \mu_b \rho g H \cos\beta(1-\lambda_b) + \rho g H \sin\beta &= (q+\lambda(1-q))\rho g H \tan(\alpha+\beta) \\ \mu_b \rho g H(1-\lambda_b) + \rho g H \beta &= (q+\lambda(1-q))\rho g H(\alpha+\beta) \\ (\alpha+\beta) &= \frac{\mu_b(1-\lambda_b)+\beta}{(q+\lambda(1-q))} \end{aligned} \tag{12}$$

It is worth noting that the real difference from the slab case (3) is the term containing the derivative of the integral of the horizontal normal stresses. Although we have ignored rotations of the principal stresses, this result is very near that of Davis et al.'s eqn 22, differing only in our use of $\lambda + q(1-\lambda)$ for their $1 + K(1-\lambda)$ where $K$ is a fairly complex term reflecting both the strength of the wedge and the rotation of the stress field; these terms significantly differ only when $\mu$ is near $\mu_b$. (The Davis et al. formulation is also more general in addressing submarine wedges, which we have ignored here). The equivalent equations from Davis et al. are

$$(\alpha+\beta)=\frac{\mu_b(1-\lambda_b)+\beta}{1+(1-\lambda)K}$$
$$\alpha=\frac{\mu_b(1-\lambda_b)+\beta}{1+(1-\lambda)K}-\beta \tag{13}$$
$$\alpha=\frac{\mu_b(1-\lambda_b)-\beta(1-\lambda)K}{1+(1-\lambda)K}$$

Values of $K$ tend to be near 2.8 for $\mu$=0.7 and 1.9 for $\mu$=0.55. Note that if $\lambda=\lambda_b$, (13) reduces to

$$\alpha=\frac{\mu_b-\beta K}{1/(1-\lambda)+K} \tag{14}$$

This solution has some interesting characteristics. The vertical and horizontal length scales fall out entirely—this solution should work for any size wedge so long as the application of the Coulomb criterion is appropriate. Note too that for a given set of physical parameters, the topographic slope angle $\alpha$ is a linear function of the dip of the decollement. As the dip decreases, the topographic slope increases

An interesting sidelight to this is that there is not a requirement that this system be in isostatic equilibrium. This isn't much of an issue, as fold and thrust belts are almost always emplaced on plates that distributed the load, producing foredeeps or oceanic trenches.

**Implications for out-of-equilibrium thrust orogens.** There are a number of direct consequences of this analysis for the evolution of fold-and-thrust belts. We first consider a few of the consequences of the last equation in (12).

First consider changes in the pore pressure of the wedge. If the basal pore pressure increases, then the righthand side of the end of (12) gets smaller and so the wedge is now too steep for its properties. This is termed being **supercritical**: there is now an excess of force from the rear being transmitted forward and so the wedge moves forward on the decollement without any internal deformation. (Technically, the force balance is now out of whack and the wedge accelerates, which presumably requires it to thin if the backstop isn't moving). If the thrust wedge grows by accretion of material at the toe, then continued accretion will bring the toe back down to a critical geometry and gradually the rest of the wedge as well.

A rather counterintuitive result is when the pore pressure increases in the wedge without any change at the base. The wedge actually grows thicker—the normal stresses (eqns. 10 and 11) are growing smaller (the wedge is weaker) and so to continue to push against the constant resistance of the base, a greater angle to the wedge is needed. This kind of a change makes the original wedge **subcritical**: there is insufficient force from the rear of the wedge to move it forward.

How do you return to critical from subcritical? A subcritical wedge simply will not move. In general, as the wedge gets moved into (or pushed by) a "backstop"—a region at the thick end of the wedge which is not part of it but acting as a sort of rigid boundary—the area at the backstop first reaches failure. As it does so, thrusts form and the wedge thickens near the backstop, eventually returning to the new critical angle. Once it does so, the forces are now transmitted forward to the region that cannot support them, and so it fails, etc. During all this, the frontal thrust of the wedge is inactive.

There are a number of changes that can drive a wedge away from a critical taper: erosion reduces $\alpha$ and so makes a wedge subcritical; subsidence of the thicker part of a wedge will tend to make a wedge subcritical; increasing $\mu$ (making the wedge stronger) will make a wedge supercritical.

**Limits of the theory**. Although failure criteria similar to the Coulomb criterion are experimentally and observationally appropriate to the upper crust, at greater depths deformation ceases to be related to pressure and this theory breaks down. These depths are only reached for individual thrust belts in a few instances (the modern Andes at the Altiplano and the Himalaya in the Tibetan Plateau). Observationally at this point the surface slope becomes near zero.

Technically, non-Coulomb behavior anywhere in the wedge would invalidate the application of the theory. Most widely mentioned are salt beds in the basal thrust that deform plastically. In practice, a low effective strength for the basal decollement allows these orogens to be examined in this context, albeit more approximately.

The “backstop” for the system also is a bit ad hoc. In practice, there is often a wedge facing the opposite direction at the other side of the backstop. A reflecting boundary condition at the left edge of the wedge would properly account for this situation; only small differences from the current solution would be present, and those mostly in the area of wedge interaction. In some instances there might actually be a strong piece of lithosphere, though this is most likely for smaller wedges.