Inverting Fault Slip Vectors

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Some relevant background is section 5.1.2 of Stüwe and 8.3-8.4 of Turcotte and Schubert.

There has long been a desire to summarize geological and seismological observations in terms of some net force or deformation that applies to the whole of the observations. These summaries can then be used to infer changes in the forces driving deformation or the style of deformation itself. The closest overview to all of this is Angelier (1994), though it is a tad biased in viewpoint.

Andersonian faulting. Perhaps most basic to this is, once again, the Coulomb failure criterion

$$|\tau| = S_0 + \mu\sigma \tag{1}$$

This criterion can be recast to say that a fault that first fails a previously isotropic rock will have a pole along the great circle between the σ_1 and σ_3 axes with a slip vector in the fault plane and σ_1-σ_3 plane. For typical rocks, the angle between the fault normal and σ_3 will be about 30°. In the early 1940s Anderson took this and the fairly simple constraint that a principal stress must be normal to the surface of the Earth to lay out what is usually called Andersonian mechanics: compression (σ_1 horizontal and σ_3 vertical) is accommodated by thrust faults dipping 30°, extension (σ_1 vertical and σ_3 horizontal) is accommodated by normal faults dipping 60°, and plane stress (σ_1 and σ_3 horizontal) is accommodated by vertical strike-slip faults.

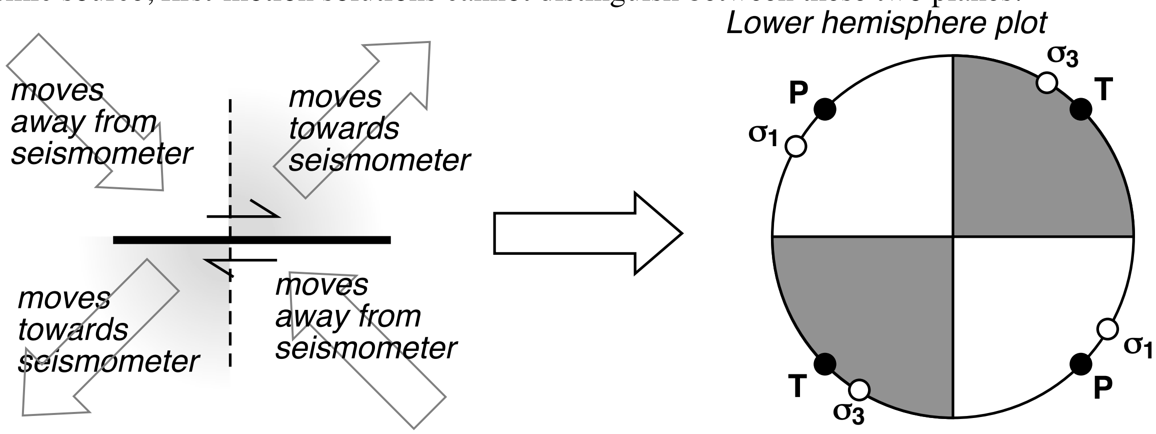
Armed with this simple formulation, geologists could argue for changes in the stress field. They could also infer the orientation of stresses if the sense of slip and paleohorizontal were known, of if conjugate faults were present. It was recognized early on that the stress field at depth could well be rotated from having a vertical component (e.g., Hafner, 1951). If conjugate faults could be recognized, then the stresses could still be inferred. Although seemingly primitive, this is still a viable option for primary fractures due to the small number of additional assumptions (e.g., Erslev, 2001).

The first wrinkle in understanding faults comes in here. Clearly Andersonian faulting cannot accommodate any deformation parallel to the intermediate stress axis, σ_2. If faults are formed in a triaxial strain system, they cannot be Andersonian (most experimental setups are uniaxial, as a piston compresses a cylinder with a metal jacket providing confining pressure). Faults that do form have lower, orthorhombic symmetry (e.g., Reches, 1987). For instance, normal faults in this situation will not strike parallel to σ_2 but will trend slightly off of it, and slip will not be pure dip-slip but instead will be somewhat oblique slip. Krantz (1989) considered some of the implications of this for some of the slickenline inversions discussed below.

Preexisting weaknesses: Wallace-Bott hypothesis. Many faults exhibit multiple phases of motion, and it seems quite likely that fractures or other weaknesses in a rock could fail before a new fault is formed in a rock. In general, one could use the Coulomb failure criterion as before, but with different values of strength S_0 and frictional coefficient μ for the plane of weakness. This quickly becomes awkward, and instead of worrying about the conditions necessary to cause slip, workers instead used the observation of faulting in a simple way: when faulting occurred, they hypothesized that the shear stress resolved onto the fault plane had to parallel the slip vector produced. This seems well supported by observational evidence. This is sometimes termed the

Wallace-Bott hypothesis after early advocates. This is a fairly fundamental assumption for most inversions of fault slip data for stress.

P and T axes and the right dihedra. At this point it is easiest to proceed by considering a common seismological observation: the earthquake focal mechanism. First arrivals from earthquakes at seismographs are either up or down. When a proper velocity structure is known for the area, the raypath from the event to the seismograph can be reconstructed. Where each raypath penetrates a small sphere centered on the earthquake, a point is plotted indicating whether the first arrival is up or down. Generally this is plotted onto a lower hemisphere stereonet (though some researchers, mostly in Europe, will use an upper hemisphere). The arrivals will separate into four quadrants (or dihedra), two where arrivals are up (or compressional) and two where they are down (dilatational). Usually the compressional quadrants are shaded. The boundaries between the quadrants are the two possible fault planes. One is the real fault plane, and the other is the auxillary plane. Because of symmetry from the seismic source, first-motion solutions cannot distinguish between these two planes.



Seismologists have long assigned names to the center of each quadrant; the center of the dilatational quadrant is termed the P axis, that of the compressional quadrant the T axis. The line where the fault and auxillary planes intersect is the B axis. If the P and T axes were σ_1 and σ_3, then the two planes would represent the planes of maximum shear stress.

This in fact is not the case generally, for even with simple Andersonian mechanics, we would predict that the principal compressional stress σ_1 would be rotated somewhat from the P axis towards the slip vector and the least compressional principal stress σ_3 would be rotated away from the slip vector, as illustrated above. However, unless we can determine the true fault plane, we don't know which way to go from the P and T axes.

This problem led seismologists into efforts to figure out how to determine the true fault plane, but it also became clear that the same Wallace-Bott hypothesis would apply here. In fact, if you presume that the earthquake occurred under that hypothesis, the loosest constraint on the stresses is that σ_1 lies in the dilatational quadrant (or P dihedra) and σ_3 lies in the compressional quadrent (T dihedra). Although this constraint has been attacked as physically unrealistic (the shear stresses resolved onto the fault plane become exceptionally small as the principal stresses near the fault planes), the complications necessary to accommodate a physically stricted model have not generally seemed warranted.

As an aside (as we are not especially considering focal mechanisms), the step of making a focal mechanism leads to uncertainty in the mechanism, which is usually ignored by the techniques discussed here (even the ones allowing uncertainty in measurements do not in general consider the specific uncertainties of a focal mechanism). Abers and Gephart (2001) addressed the joint uncertainties in using focal mechanisms for stress inversion.

Note that we can construct these quadrants from any kind of fault data. The auxillary plane is normal to the slip vector and the sense of slip determines the assignment of P and T quadrants.

Simplest use of dihedra. It may be seen that if we have multiple slip measurements, we could simply superimpose several of the shaded lower-hemisphere plots to determine the places where the principal stresses must fall. This was in fact one of the earlier approaches used in combining larger collections of fault slip measurements. It becomes flawed when measurements are discrepant. A somewhat similar approach favored by seismologists is to contour the P and T axes and use the contour peaks as measures of principal stresses. This is, however, not as rigorous.

Resolving stress. To continue, we need to be able to know the direction and magnitude of stress on an arbitrary plane. One solution to rotating the stress tensor provided by Jaeger and Cook (*Fundamentals of Rock Mechanics, sec 2.5*) uses the direction cosines between the principal stress axes and a new set of axes:

$$\sigma_{x'} = l_1^2\sigma_1 + m_1^2\sigma_2 + n_1^2\sigma_3 \quad (1)$$

$$\sigma_{y'} = l_2^2\sigma_1 + m_2^2\sigma_2 + n_2^2\sigma_3 \quad (2)$$

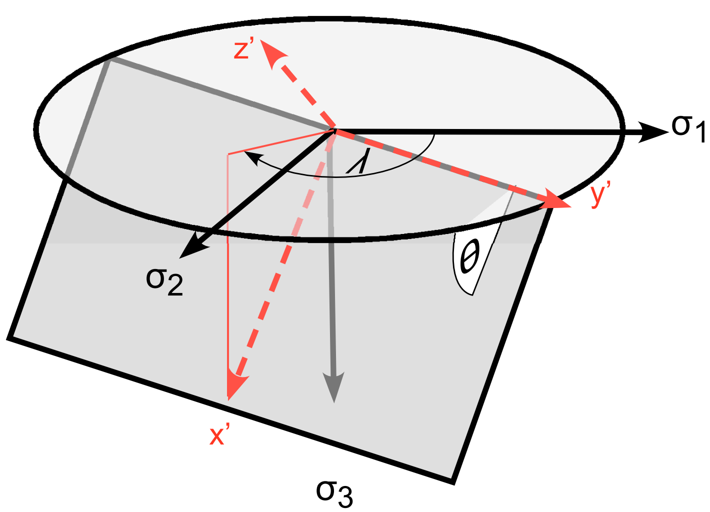
$$\sigma_{z'} = l_3^2\sigma_1 + m_3^2\sigma_2 + n_3^2\sigma_3 \quad (3)$$

$$\tau_{x'y'} = l_1 l_2\sigma_1 + m_1 m_2\sigma_2 + n_1 n_2\sigma_3 \quad (4)$$

$$\tau_{y'z'} = l_2 l_3\sigma_1 + m_2 m_3\sigma_2 + n_2 n_3\sigma_3 \quad (5)$$

$$\tau_{z'x'} = l_3 l_1\sigma_1 + m_3 m_1\sigma_2 + n_3 n_1\sigma_3 \quad (6)$$

where the l's are the direction cosines of the x', y', and z' axes with the x (σ_1) axis, m's with the y (σ_2) axis, and n's with the z (σ_3) axis. (Direction cosines are the cosines between the two axes, which is equal to the dot product of unit vectors parallel to those axes). While mathematically simple and useful in many applications (especially once you recognize the underlying matrix math), this is more than we might want for simply knowing the stresses on a plane.



Consider a case where σ_1 and σ_2 to lie in the horizontal plane and we wish to know the slip resolved onto a plane with a dip of θ and a dip direction of λ measured from the x (σ_1)-axis towards the y-axis. Let x' point downdip and then y' is in the σ_1-σ_2 plane as the strike (face towards positive y' and the plane dips to the right). Then we only want σ_z and $\tau_{z'x'}$ and $\tau_{y'z'}$ to know the stresses on the plane. So in the (x,y,z) frame of the principal stresses, $x' = (l_1,m_1,n_1) = (\cos(\lambda)\cos(\theta),\sin(\lambda)\cos(\theta),\sin(\theta))$, $y' = (l_2,m_2,n_2) = (\sin\lambda,-\cos\lambda,0)$ and $z' = (l_3,m_3,n_3) = (\cos(\lambda)\sin(\theta),\sin(\lambda)\sin(\theta),-\cos(\theta))$. In this case, the normal and shear stress components on this plane are

$$\sigma_{z'} = \left(\sigma_1\cos^2\lambda + \sigma_2\sin^2\lambda\right)\sin^2\theta + \sigma_3\cos^2\theta \tag{7}$$

$$\tau_{y'z'} = \frac{1}{2}\left(\sigma_1 - \sigma_2\right)\sin\theta\sin 2\lambda \tag{8}$$

$$\tau_{x'z'} = \frac{1}{2}\left(\sigma_1\cos^2\lambda + \sigma_2\sin^2\lambda - \sigma_3\right)\sin 2\theta \tag{9}$$

These directions may then be combined to find the rake α of the maximum shear stress on the plane:

$$\begin{aligned}
\alpha &= \arctan\frac{\tau_{x'z'}}{\tau_{y'z'}} \\
&= \arctan\frac{\left(\sigma_1\cos^2\lambda + \sigma_2\sin^2\lambda - \sigma_3\right)\sin 2\theta}{\left(\sigma_1 - \sigma_2\right)\sin\theta\sin 2\lambda} \\
&= \arctan\frac{2\left(\sigma_1\cos^2\lambda + \sigma_2\sin^2\lambda - \sigma_3\right)\cos\theta}{\left(\sigma_1 - \sigma_2\right)\sin 2\lambda} \\
&= \arctan\frac{2\left(\left(\sigma_1 - \sigma_3\right)\cos^2\lambda + \left(\sigma_2 - \sigma_3\right)\sin^2\lambda\right)\cos\theta}{\left(\sigma_1 - \sigma_2\right)\sin 2\lambda} \\
&= \arctan\frac{2\left(\cos^2\lambda + \dfrac{\left(\sigma_2 - \sigma_3\right)}{\left(\sigma_1 - \sigma_3\right)}\sin^2\lambda\right)\cos\theta}{\dfrac{\left(\sigma_1 - \sigma_2 + \sigma_3 - \sigma_3\right)}{\left(\sigma_1 - \sigma_3\right)}\sin 2\lambda} \\
&= \arctan\frac{2\left(\cos^2\lambda + \Phi\sin^2\lambda\right)\cos\theta}{\left(1 - \Phi\right)\sin 2\lambda}
\end{aligned} \tag{10}$$

where $\Phi = (\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)$. Trivial substitutions produce similar solutions with the orientations of the principal stresses permuted over the three axes so long as one is normal to the free surface. From (10) it is evident that the direction of shear on a plane depends only on the orientation with respect to the principal stresses and Φ. Similarly it can be shown that the magnitude of the shear stress depends on these parameters and $(\sigma_1-\sigma_3)$; the absolute magnitude of stresses is harder to obtain.

Stress inversion. As data became more numerous, the problem quickly became overdetermined: there were far more observations than model parameters to determine. This lends itself to inversion. The forward problem is fairly simple: for each fault plane, determine the slip direction given a specified stress field. The inversion, then, is fairly straightforward (though more than we can to deal with here). The trick, however, is to settle on what function we seek to minimize when doing the inversion. Most initial inversions minimized the misfit of the predicted shear and observed slip within the fault plane. Later modifications (e.g., Angelier et al., 1982, Gephart and Forsyth, 1984) included error in the fault plane as well. A noticeably different criterion was to maximize $|\lambda \mathbf{s} \bullet \boldsymbol{\tau}|^2$, where **s** is the observed slip vector and **τ** is the shear stress resolved on the best-fitting fault plane (Angelier, 1990). This formulation rewards stress fields that increase the magnitude of the shear stress as much as matching the direction.

The exact type of misfit function can be important. Many of the inversions use a least-squares norm for the fitting function; this is equivalent to assuming a Gaussian distribution of the residuals. This is generally not the case, and somewhat less sensitivity to outliers is desired. Angelier (1994) advocates removal of anomalous data by hand. Alternatives include the use of non-least squares (non-L2 norm) misfit functions (e.g., Gephart, 1990) and explicitly spherical probability distributions (e.g., Yin and Ranalli, 1993).

As long as fitting the direction of slip is considered, though, the full stress tensor cannot be recovered. The direction of resolved shear stress on a plane is independent of the scalar magnitude of the stresses and of any isotropic stress added to the stresses. That is, the principal stresses

$$\begin{bmatrix} \sigma_1' & 0 & 0 \\ 0 & \sigma_2' & 0 \\ 0 & 0 & \sigma_3' \end{bmatrix} = \begin{bmatrix} a\sigma_1 & 0 & 0 \\ 0 & a\sigma_2 & 0 \\ 0 & 0 & a\sigma_3 \end{bmatrix} + \begin{bmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix} \qquad (2)$$

will satisfy the solution. In practice, this means that two of the six independent elements of the stress tensor are undetermined; the four that are solved are the orientation of the principal stresses and $\Phi = (\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)$. The part of the stress tensor usually resolved is termed the reduced stress tensor.

Fully recovering the stress tensor requires additional assumptions. Some attempts have included assuming that all the faults slip with the same applied shear stress (Michael, 1984) and that all the faults behave according to Coulomb failure (Reches, 1987b; Yin and Ranalli, 1995). Assumptions about lithostatic stresses and the maximum difference in stress that rocks can support can also limit the stress tensor (e.g., Yin, 1996). For many workers, these additional assumptions have not yielded significantly more useful results.

Assumptions in stress inversions. The two most obvious assumptions in making stress inversions are that the stress is uniform and invariant over time. In both cases workers generally propose that systematic misfits can identify areas that violate assumptions about homogeneity (as with Angelier’s suggestion in identifying outliers and removing them, by hand or algorithm). Bootstrap-style resampling approaches can also be applied to this problem (Albarello, 2000.). Some of these effects are explicitly examined in a forward sense by Pollard et al. (1993), who find that stress variations from nearby large faults can bias a result away from a regional stress field, but this itself does not seem to invalidate the inversion for the overall stress field of the area under study.

More fundamental is the assumption of infinitesimal strain, which is related to the assumption that the shear stress parallels slip. In fact, fault slip introduces a finite strain. Twiss and Unruh (1998) argue that the rotational component of the finite strain invalidates most inversions for stress. They claim that these inversions are more apt to recover some measure of the principal strains and will only recover the stresses when there is no internal rotation of material (roughly equivalent to pure shear deformation). They argue that the mean P and T axes are in fact good measures of the finite shortening and lengthening of a body but can be poor proxies for stress. They prefer an alternate methodology, micropolar analysis, which incorporates rotations of very small elements within a region deforming as a continuum; the analogy that underlies this theory is to cataclastic flow, where material is spinning about as shear across the system is accommodated. In this analysis, slip orientations are parallel not to the maximum resolved shear stress, but to the maximum resolved shear rate from a large-scale strain rate tensor, with modifications for rotations of blocks within the flow.

A poorly addressed issue, especially in field studies, is bias from available exposure. In some instances, exposures that are examined necessarily have only smaller offset faults. Larger faults might be known from nearby but cannot be included due to lack of exposure or other complications. In these instances, interpretations of stress fields must be regarded with care as Pollard et al. (1993) have shown systematic biases to be possible. It can also be possible that a structurally anisotropic, inhomogeneous body of rock will both tend to preserve fault data only within more competent strata that might have stress concentrations and rotations due to the inhomogenieties. These biases might not be revealed in the misfits to the available striations.

A more recent overview of potential biases and difficulties in applying stress inversions was made by Lecombe (2012) and follows a discussion of difficulties in applying this approach made by Sperner & Zweigel (2010). Sperner and Zweigel focus on difficulties mentioned above (identifying slip on all relevant faults, correctly deducing temporal variations, and inappropriate grouping of observations). Lecombe takes a broader view and includes criticism of some of the seminal assumptions in different approaches. Both papers fault some researchers for treating these analyses as black boxes. On a related note, Lejri et al. (2015) explore the conditions where the Wallace-Bott assumption of resolved shear stress on a fault paralleling slip is violated.

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