GEOL5690 Class notes #2: Passive Margins & Thermal Subsidence

Reference: Turcotte and Schubert, sec. 4-15, 4-16, 4-23, 4-25, last is most relevant, earlier readings provide background. [Alternative: Stüwe, sections 6.1.3 & 6.1.4; this also includes a description of how backstripping works missing from Turcotte and Schubert]

Bond, G. C., and M. A. Kominz, Construction of tectonic subsidence curves for the early Paleozoic miogeocline, southern Canadian Rocky Mountains; implications for subsidence mechanisms, age of breakup, and crustal thinning, Geological Society of America Bulletin, 95, (2), 155-173, 1984. Chs. 2 and 3 of Baldridge are relevant.

Overall, this is relatively straightforward and quite elegant. Passive margins are the edges of continents, going from undeformed cratons out into fresh seafloor. Distinctive thick wedges of sedimentary rock are deposited on these margins, and the ability to interpret these sedimentary packages for the tectonic history is our goal.

Conduction of heat

How is heat moved in the Earth? We are all familiar with conduction of heat. Conductive heat flow q is related to the temperature gradient: $q=-k\frac{dT}{dz}$, where T is the temperature with depth and k is the coefficient of thermal conductivity (the minus sign reflects heat going from hot to cold and often is omitted in the simple 1-D case but becomes important in three dimensions). For typical crustal rocks, k is 2-3 W m^{-1} °C^{-1}, though this increases somewhat to 3-5 W m^{-1} °C^{-1} for dunite and peridotite. Most heat is conducted out of the Earth through conduction (some is by fluid flow, which is a separate problem). The average heat flow in ocean basins is 101 mW/m^2, continents 65 mW/m^2, and the global average is 87 mW/m^2 for a global total heat loss of about 4.43 x 10^{13} W. In steady state, without a change in temperature boundary conditions, temperatures should vary linearly.

A simple application of this is the cooling of a warm body. If the body has material with a specific heat of C (the specific heat per unit mass, in units of J kg^{-1} °C^{-1}) and a density ρ, then a decrease in temperature of ΔT represents the loss of $(\Delta T)\rho C$ J/m^3 of heat. If it is radioactive, it will generate heat at a rate of A per unit volume per unit time. The loss (or gain) of heat is the difference between the heat flow in and that out, which occurs at a rate of $\frac{dq}{dz}$, so we may write

$$\frac{dq}{dz}=A-\frac{d}{dt}(\rho CT) \qquad (2.1)$$

where the minus reflects the fact that a positive change in heat flow results in a negative change in temperature. By using the relationship between heat flow and conduction, (2.1) can become

$$k\frac{d^2T}{dz^2} = \rho C\frac{dT}{dt}$$
$$\frac{dT}{dt} = \kappa\frac{d^2T}{dz^2} \qquad (2.2)$$

where κ = $k/\rho C$ and is termed the thermal diffusivity.

Thermal structure of cooling (or heating) halfspace

Let's take the extreme case of freshly minted ocean floor and use our understanding of heat conduction to see how it should evolve in elevation. We will largely be paralleling discussions developed in Turcotte and Schubert before veering off into basins with some continental crust and lithosphere left. We will make the problem quite easy by assuming that heat is lost through the top of the oceanic lithosphere and not the sides. We start then with our simple equation for one dimensional change in temperature for material not producing heat internally.

$$\frac{dT}{dt} = \kappa\frac{d^2T}{dz^2} \qquad (2.3)$$

Let us specify the boundary conditions. At the start (t=0), the temperature in the halfspace will be T_a, and at the surface (z=0) it will be T_0. We'll also specify that the temperature at great depth never changes, so at $z=\infty$ we'll force the temperature to be T_a. Given these boundaries, we expect that temperatures will always be between T_0 and T_a, so a dimensionless variable indicating the fraction of the temperature difference will prove helpful:

$$\theta = \frac{T - T_a}{T_0 - T_a} \qquad (2.4)$$

It can be shown that equation (2.3) is essentially unchanged by the substitution of variables:

$$\frac{d\theta}{dt} = \kappa\frac{d^2\theta}{dz^2} \qquad (2.5)$$

except that the boundary conditions are now θ at the surface is 1, at time 0 it is 0, and at infinity it is 0. The only part of the problem suggesting a length scale is the thermal diffusivity, which is length^2/time, so a practical approach is to try and make the problem totally nondimensional by substituting the dimensionless parameter

$$\eta = \frac{z}{2\sqrt{\kappa t}} \qquad (2.6)$$

We differentiate this with respect to t and z to get the proper terms for (2.5):

$$\frac{\partial\theta}{\partial t}=\frac{d\theta}{d\eta}\frac{\partial\eta}{\partial t}=\frac{d\theta}{d\eta}\left(-\frac{1}{2}\frac{\eta}{t}\right)$$
$$\frac{\partial\theta}{\partial z}=\frac{d\theta}{d\eta}\frac{\partial\eta}{\partial z}=\frac{d\theta}{d\eta}\frac{1}{2\sqrt{\kappa t}} \tag{2.7}$$
$$\frac{\partial^2\theta}{\partial z^2}=\frac{1}{2\sqrt{\kappa t}}\frac{d^2\theta}{d\eta^2}\frac{\partial\eta}{\partial z}=\frac{1}{4\kappa t}\frac{d^2\theta}{d\eta^2}$$

which, when placed back into (2.5) yields

$$-\eta\frac{d\theta}{d\eta}=\frac{1}{2}\frac{d^2\theta}{d\eta^2} \tag{2.8}$$

and the boundary conditions have reduced to $\theta(\infty)=0$ and $\theta(0)=1$. We first solve for $\frac{d\theta}{d\eta}$ which will will term ϕ, so (2.8) becomes

$$-\eta\phi=\frac{1}{2}\frac{d\phi}{d\eta} \tag{2.9}$$

which is easily integrated to find that

$$-\eta^2=\ln\phi-\ln c_1 \tag{2.10}$$

where c_1 is a constant of integration. If we take the exponential of both sides we get

$$\phi=c_1e^{-\eta^2}=\frac{d\theta}{d\eta} \tag{2.11}$$

which can also be integrated to solve for θ:

$$\theta=c_1\int_0^{\eta}e^{-\eta'^2}d\eta'+c_2 \tag{2.12}$$

Applying our boundary conditions, when $\eta = 0$ θ should be 1, and as the integral from 0 to 0 is 0, c_2 must be 1. When $\eta = \infty$ the integral becomes one commonly known to be $\frac{\sqrt{\pi}}{2}$ and so therefore c_1 must be $-\frac{2}{\sqrt{\pi}}$, which leads us to the solution

$$\theta=1-\frac{2}{\sqrt{\pi}}\int_0^{\eta}e^{-\eta'^2}\,d\eta' \tag{2.13}$$

This is a common enough integral that it has it's own name, the error function, erf(η) and so we rewrite our solution as

$$\theta = 1 - \operatorname{erf}(\eta) = \operatorname{erfc}\eta \tag{2.14}$$

where erfc is the complementary error function. We can substitute our original variables back in to get

$$\frac{T - T_a}{T_0 - T_a} = \operatorname{erfc}\frac{z}{2\sqrt{\kappa t}} \tag{2.15}$$

What this tells us is that the depth of an isotherm increases with $\sqrt{\kappa t}$. A simple rearrangement of (2.15) is also handy:

$$1 - \frac{T - T_a}{T_0 - T_a} = \frac{T_0 - T}{T_0 - T_a} = \frac{T - T_0}{T_a - T_0} = \operatorname{erf}\frac{z}{2\sqrt{\kappa t}} \tag{2.16}$$

As the cooling seafloor represents the top of the convecting mantle, it is sometimes helpful to define the thermal boundary layer from this conductive solution even though such a definition is inherently vague. Turcotte and Schubert define the thickness of the thermal boundary layer as θ = 0.1. Plugging this into (2.14), we find the equivalent value of η to be 1.16, so from (2.6) the thickness is $2.32\sqrt{\kappa t}$. (Similarly, the temperature on the surface at the depth of the thermal diffusion length $\sqrt{\kappa t}$ is η = 0.5, which yields θ = 0.4795, which is about halfway from the original temperature to the surface temperature).

Subsidence of seafloor

If we assume that all variations in topography of oceanic lithosphere are due to temperature variations (also called Pratt isostasy sometimes, after an early advocate of supporting topography through variations in densities), we can directly use our analysis to get at variations in topography. This is not a bad assumption, for most seafloor crust is generated very near the ridgecrest and is not modified much over its whole history. In fact, by removing the thermal variations you can see these other effects more clearly. So let us assume local isostasy. The condition for local isostasy is that the pressure at some reference depth is the same everywhere. The pressure in turn is the integral of the weight of all the material above:

$$P_c = \int_0^{z_c} \rho g\, dz = dg\rho_w + \int_0^{z_c'} \rho g\, dz' \tag{2.17}$$

where P_c is the pressure at the depth of compensation, which is z_c below sea level or z'_c below the seafloor. We have *d* as the depth of the water. If we now set the pressure under a column at the midocean ridge at a water depth of *d* equal that under a column elsewhere at depth *w*+*d* of age *t*, we get

$$d_{ridge} g\rho_w + \int_d^{z_c} \rho g\, dz = \left(d_{ridge} + w\right) g\rho_w + \int_{d_{ridge}+w}^{z_c} \rho g\, dz$$
$$\int_{d_{ridge}}^{d_{ridge}+w} \rho_a\, dz + \int_{d_{ridge}+w}^{z_c} \rho_a\, dz = w\rho_w + \int_{d_{ridge}+w}^{z_c} \rho(z)\, dz \qquad (2.18)$$
$$w\left(\rho_a - \rho_w\right) = \int_{d_{ridge}+w}^{z_c} \left(\rho(z) - \rho_a\right) dz$$

We can use the coefficient of thermal expansion to relate our density to the temperature, $\rho - \rho_a = -\rho_a \alpha \left(T - T_a\right)$, and then use the temperature from (2.15), converting our limits of integration to now be downward from the seafloor of the old ocean floor to the depth of compensation under that seafloor:

$$\begin{aligned} w\left(\rho_a - \rho_w\right) &= \int_0^{z_c'} -\rho_a \alpha \left(T_0 - T_a\right) \operatorname{erfc} \frac{z'}{2\sqrt{\kappa t}} dz' \\ &= \rho_a \alpha \left(T_a - T_0\right) \int_0^{\infty} \operatorname{erfc} \frac{z'}{2\sqrt{\kappa t}} dz' \end{aligned} \qquad (2.19)$$

We again allowed the integral to go to infinity because the temperatures converge below z'_c. We repeat the substitution used in (2.6) so that the integral is more simply dealt with:

$$\begin{aligned} w &= \frac{\rho_a \alpha \left(T_a - T_0\right)}{\rho_a - \rho_w} 2\sqrt{\kappa t} \int_0^{\infty} \operatorname{erfc} \eta\, d\eta \\ &= \frac{2\rho_a \alpha \left(T_a - T_0\right)}{\rho_a - \rho_w} \sqrt{\frac{\kappa t}{\pi}} \end{aligned} \qquad (2.20)$$

where we have again used a standard result for the integral of the complementary error function. This is the desired result that expresses the depth of the seafloor relative to the ridge as the square root of the age of that sea floor. If spreading is constant, then the time can be replaced by the distance from the ridge over the half spreading rate. Note that this solution ignores the presence of any sediments on the ocean floor.

Basin subsidence

OK, that gets the thermal subsidence down easily for ocean floor, but how does that relate to changes in older continental crust? To make room for many kilometers of sediment, isostasy demands we make some room. This can be done in two main ways: one is to have a lengthy enough thermal uplift that erosion removes the uppermost crust; the other is through extensional thinning of the crust. The latter was proposed by McKenzie (1978) to understand the subsidence of the North Sea and other basins.

The concept is very similar to ocean floor subsidence to the degree that Turcotte and Schubert advocate using (2.20)(above) with the minor change of sediment density for water density. This is probably too crude for real application, as the starting condition is not a uniform halfspace with asthenospheric temperature but instead some attenuated lithospheric section with a compressed geotherm. Stüwe has the subsidence from the rifting phase as

$$w_{rift} = z_c\left(\frac{\rho_a - \rho_c}{\rho_a - \rho_f}\right)\left(1-\frac{1}{\delta}\right) - z_l\left(\frac{\rho_a \alpha T_a}{2\left(\rho_a - \rho_f\right)}\right)\left(1-\frac{1}{\beta}\right) \tag{2.21}$$

(removing some STP values as probably incorrect) but my version is:

$$w_{rift} = z_c\left(\frac{\rho_a - \rho_c}{\rho_a - \rho_f}\right)\left(1-\frac{1}{\delta}\right) - z_l\left(\frac{\rho_m - \rho_a}{\rho_a - \rho_f}\right)\left(1-\frac{1}{\beta}\right) \tag{2.22}$$

where β and δ are the thinning factors (stretching factors) for mantle lithosphere and crust, respectively (so uplift is possible if β greater than δ by some amount). Note that we've allowed the sinking crust to be covered either by water (ρ_f = 1000 kg m^{-3}) or sediment (ρ_f ~ 2300 kg m^{-3}). Stüwe's equation assumes that the mean temperature of the mantle lithosphere is half the asthospheric temperature and so will tend to overpredict subsidence. Initial thickness of crust and mantle lithosphere are z_c and z_l. (Stretching factor = final width/initial width, or initial thickness/final thickness for plane strain; it is the thickness that matters for these calculations). These calculations assume that this happened quickly enough that we can ignore any thermal evolution of the lithosphere while stretching occurs (this is known as instantaneous stretching). Roughly speaking, we can ignore the evolution of rifting if it takes less than ~15 My for a thinning of a factor of 2 and less than 60 My for infinite stretching.

We then consider the thermal reequilibration of the lithosphere. Ignoring the cooling of the crust as a significant factor in the subsequent thermal subsidence, the subsequent thermal subsidence of the lithosphere can be represented (6.11 of Stüwe) as

$$w_{sag} = \left(\frac{4\rho_a \alpha T_a z_a}{\pi^2\left(\rho_a - \rho_f\right)}\right)\left(\frac{\beta}{\pi}\sin\left(\pi/\beta\right)\right)\left(1-e^{-t/\tau}\right) \tag{2.23}$$

where the final equilibrium depth of the base of the lithosphere is z_a and the time scale of thermal equilibrium is $\tau = z_l^2/(\pi^2\kappa)$. Again, ρ_f is the material filling the newly-created accommodation space; the presence of this term (just as it shows up for subsidence of the ocean floor) represents the isostatic adjustment depending on the fill. This is an approximation appropriate for some significant time after initial rifting as it is just using the first term of a Fourier solution.

Note that while the total magnitude of the subsidence depends on the stretching factor, the e-folding time (decay timescale) does not. Also note that changing the material filling the accommodation space; if filled with sediment of a uniform density, the magnitudes increase relative to air- or water-filled basins but again the shapes of the curves remain the same. Although the result McKenzie got doesn't have the square root of time relationship, it does closely approximate it out to about 50-80 My (depends on the extension amount and z_l).

Backstripping and Tectonic subsidence

Refs: Bond and Kominz, 1984; Steckler and Watts, 1978, EPSL 41, 1-13

If we knew how the basement to a basin subsided were there no sediments, we would have something we could directly match to the equations above. The problem is that the margins of continents are covered with sediments (this is also an advantage as this gives us a history through time). The trick is to remove the sedimentary rocks and figure out where the basement would have been without them; this is termed the **tectonic subsidence**. Because sedimentary rocks compress under weight, this is not entirely trivial. First, though, consider how sedimentary subsidence works: imagine basement is sitting at sea level to start with. We add a unit of thickness t_1 and density ρ_1 onto the basement; the weight of this rock ($t_1\rho_1$) will cause basement to subside through isostasy:

$$s(1) = t_1 \frac{\rho_1 - \rho_w}{\rho_a - \rho_w} \tag{2.24}$$

This s is termed the **sedimentary subsidence**. The **total subsidence** is the sedimentary subsidence plus the tectonic subsidence and should be where basement was relative to sea level over time. The accumulation of sediment will cause the surface of the rock column (seafloor) to rise up to a depth

$$w(1) = w(0) + s(1) - t_1 = w(0) - t_1 \frac{\rho_a - \rho_1}{\rho_a - \rho_w} \tag{2.25}$$

where $w(t)$ is the water depth at time reference t and ignoring changes in sea level and overlooking any tectonic changes. Now if in fact the top of the unit was deposited at a depth $w_o(1)$, then the tectonic subsidence over this time interval would be $w_o(1) - w(1)$. If we had a stack of sediments that were incompressible, we could simply do this straightforward calculation and have a curve of both the total subsidence and the tectonic subsidence. Unfortunately, sediments do compact rather strongly.

Compaction

Most of the compaction of sediments is from the reduction in pore space. Observationally this is found to approximate an exponential decay in porosity with depth:

$$\phi(z) = \phi_0 e^{-kz} \tag{2.26}$$

where k and ϕ_0 depend upon the lithology. For instance, Stüwe suggests

Lithology	ϕ_0	k	ρ_g
Sandstone	0.4	$3 \cdot 10^{-4}$ m^{-1}	2650 kg m^{-3}
Limestone	0.5	$7 \cdot 10^{-4}$ m^{-1}	2710 kg m^{-3}
Slate (shale)	0.5	$5 \cdot 10^{-4}$ m^{-1}	2720 kg m^{-3}

where ρ_g is the density of the grains, so the mean density of the rock is

$$\rho = \rho_g(1-\phi) + \rho_w\phi \tag{2.27}$$

where ρ_w is the density of pore fluid (usually water). In order to properly recover the subsidence, we have to restore our pile of sediments. Doing this from an observed pile of sediment is termed **backstripping**. The basic assumption is that the change in thickness of sediments is due mainly to compaction (reduction in pore space). In some sections, cementation and dissolution are significant and have to be considered (styolites, for instance, indicate dissolution). If we have a pile of sediments of current thickness L, and we are interested in the thickness L_i at the time a horizon i at modern depth z_i was deposited, then we simply equate the amount of rock in each column:

$$\int_0^{L_i} (1-\phi)\,dz = \int_{z_i}^{L} (1-\phi)\,dz \tag{2.28}$$

This can be integrated but only solved numerically from the equation above. In practice, because you have varying lithologies, workers break the section down into individual packets. So imagine horizon i has a modern thickness t_i that is equal to $t_i(z_i)$. We can approximate the original thickness of this layer $t_i(0)$ as

$$\begin{aligned} t_i(0) &= \frac{(1-\phi)}{1-\phi_0} t_i \\ &= \frac{\left(1-\phi_0 e^{-kz_i}\right)}{1-\phi_0} t_i \end{aligned} \tag{2.29}$$

Put in reverse, we can now estimate the thickness for any depth z:

$$\begin{aligned} t_i(z) &= \frac{(1-\phi_0)}{1-\phi} t_i(0) \\ &= \frac{(1-\phi_0)}{1-\phi_0 e^{-kz_i}} t_i(0) \end{aligned} \tag{2.30}$$

From this, we can iteratively build our subsidence history. If we have already figured out the subsidence when horizon *i-1* was deposited, then we can estimate changes in the thickness of the sedimentary pile. Interestingly, if the entire pile were a single lithology, the the increase in thickness as $t_i(0)$ was deposited is, in fact, about $t_i(L_{i-1})$ (you can think of this as adding at the bottom of the pile since everything moves down). It represents a new load per unit area of $g\rho_g(1-\phi_0)t_i(0)$ which should produce sedimentary subsidence of $s_i = (1-\phi_0)t_i(0)(\rho_g - \rho_w)/(\rho_a - \rho_w)$ (combining (2.27) and (2.24)). The sediment surface should rise by

$$\begin{aligned} t_i(L_{i-1}) - s_i &= t_i\left(L_{i-1}\right) - \left(1-\phi_0\right) t_i(0) \frac{\rho_g - \rho_w}{\rho_a - \rho_w} \\ &= \left[\frac{1}{1-\phi_0 e^{-kL_i}} - \frac{\rho_g - \rho_w}{\rho_a - \rho_w}\right] \left(1-\phi_0\right) t_i(0) \end{aligned} \tag{2.31}$$

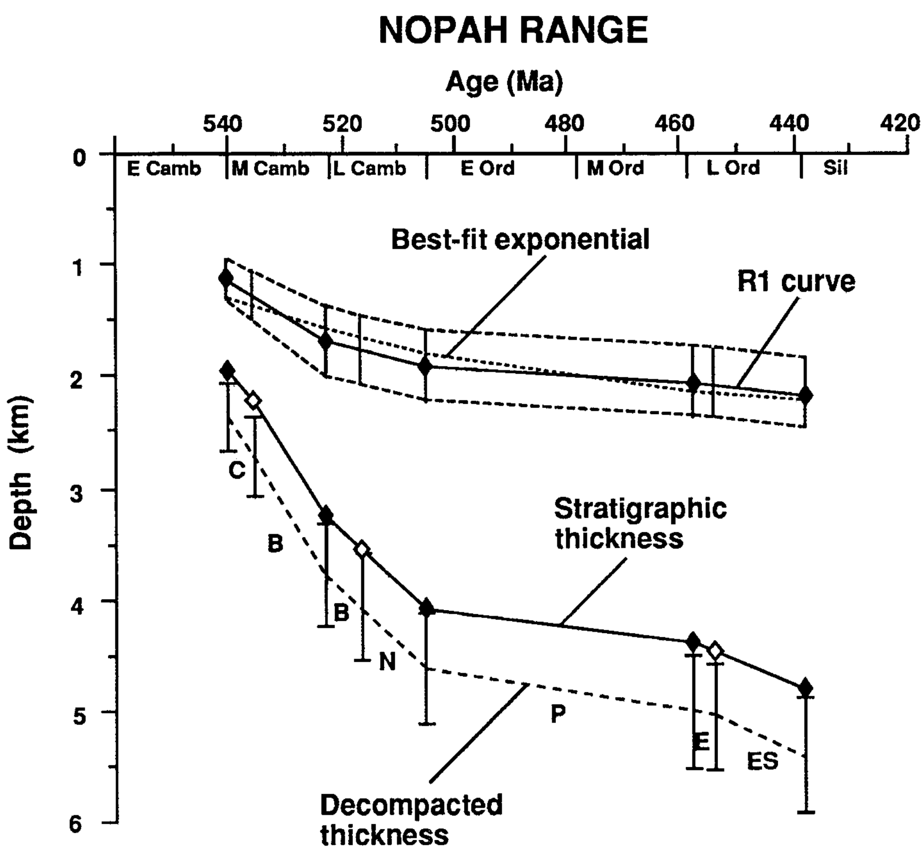
Thus the rate (per unit thickness of sediment) at which the sea is filled decreases with time. At the extreme limit where the porosity at the bottom of the sediment pile is effectively 0, adding more sediment results in the water depth decreasing by

$$\left[\frac{\rho_a - \rho_g}{\rho_a - \rho_w}\right](1-\phi_0)t_i(0) \qquad (2.32)$$

which is essentially the uplift underwater of thickening the crust.

It is worth recognizing that three processes are active in a thermally subsiding basin: First, there is isostatically-supported subsidence caused by the increasing density of the lithosphere as it cools, there is the accumulation of sediments, which drives subsidence, and there is the mechanical and chemical compaction of those sediments as they pile up. Our analysis above simply assumes that compaction and isostasy are effectively instantaneous. How true is this? Well, we have a good sense of the timescale of isostasy from post-glacial rebound. There is an elastic component that is instantaneous and not too important. Then there is the visco-elastic response, which has an e-folding time of ~5000 years and observed rates in the cm/yr range. In contrast, the rate of subsidence of thinned crust is no more than about 100m/10 My, or 0.01 mm/yr, some 1000 times slower than isostasy. Compaction is similarly fast to isostasy (some buildings built on compressible bogs subside within years; coastal Louisiana has subsided at a rate of about 2 mm/yr, with the compaction contribution estimated between 1-10 mm/yr. So for purposes of modeling sediment thicknesses, we can assume that compaction and isostasy are instantaneous.

Application of the equations above to estimate the tectonic subsidence is straightforward if numerically tedious. At any time *i* the thickness of sedimentary layers older than that time are calculated using (2.30) working from the top down. The sum of those thicknesses and the water depth (estimated by the facies of the uppermost sediments) is the total subsidence at that time (assuming that the basement was at sea level to start with). This is corrected for the subsidence due to the sediments using an equation similar to (2.24) using the thicknesses and densities appropriate at this time; this corrected value is the tectonic subsidence. Plots of these versus time are tectonic subsidence curves. An example from Levy and Christie-Blick (GSA Bulletin, 1991) below shows how the tectonic subsidence might slow substantially as sediment continues to accumulate:


NOPAH RANGE
Age (Ma)
540
520
500
480
460
440
420
0
E Camb
M Camb
L Camb
E Ord
M Ord
L Ord
Sil
Best-fit exponential
1
R1 curve
2
Depth (km)
3
C
B
Stratigraphic thickness
4
B
N
5
P
E
ES
6
Decompacted thickness