# Mellor-Yamada Level 2.5 Turbulence Closure in RAMS

Nick Parazoo

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# Overview

- Derive level 2.5 model from basic equations
- Review modifications of model for RAMS
- Assess sensitivity of vertical eddy diffusivities to tunable coefficients
- Feasibility of lookup table for RAMS

# Goal of Mellor and Yamada

- Establish a hierarchy of turbulent closure models for planetary boundary layers by the following method
  1. Obtain prognostic equations for the variance and covariance of the fluctuating components of velocity and potential temperature
  2. Simplify higher level equations according to the number of isotropic components to retain, degree of computational efficiency
  3. Introduce empirical constants that cover the most appropriate scales of turbulence

# The Basic Equations

$$\frac{\partial U_i}{\partial x_i}=0, \quad (1)$$

$$\frac{\partial U_j}{\partial t}+\frac{\partial}{\partial x_k}(U_k U_j+\overline{u_k u_j})+\epsilon_{jkl} f_k U_l = -\frac{\partial P}{\partial x_j}-g_j\beta\Theta+\nu\nabla^2 U_j, \quad (2)$$

$$\frac{\partial \Theta}{\partial t}+\frac{\partial}{\partial x_k}(U_k\Theta+\overline{u_k\theta})=\alpha\nabla^2\Theta, \quad (3)$$

$$\frac{\partial u_i}{\partial x_i}=0, \quad (4)$$

$$\frac{\partial u_j}{\partial t}+\frac{\partial}{\partial x_k}(U_k u_j+U_j u_k+u_k u_j-\overline{u_k u_j})+\epsilon_{jkl} f_k u_l = -\frac{\partial p}{\partial x_j}-g_j\beta\theta+\nu\nabla^2 u_j, \quad (5)$$

$$\frac{\partial \theta}{\partial t}+\frac{\partial}{\partial k_k}(\Theta u_k+U_k\theta+u_k\theta-\overline{u_k\theta})=\alpha\nabla^2\theta. \quad (6)$$

- Derive prognostic equations for Reynolds stress and heat conduction moments by combining equations for the mean and fluctuating components

# Governing Equations

$$\frac{\partial \overline{u_i u_j}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{u_i u_j}+\overline{u_k u_i u_j}-\nu\frac{\partial}{\partial x_k}\overline{u_i u_j}\right]+\frac{\partial}{\partial x_j}\overline{p u_i}+\frac{\partial}{\partial x_i}\overline{p u_j}+f_k(\epsilon_{jkl}\overline{u_l u_i}+\epsilon_{ikl}\overline{u_l u_j})$$
$$=-\overline{u_k u_i}\frac{\partial U_j}{\partial x_k}-\overline{u_k u_j}\frac{\partial U_i}{\partial x_k}-\beta(g_j\overline{u_i\theta}+g_i\overline{u_j\theta})+\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}-2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}, \quad (7)$$

$$\frac{\partial \overline{u_j\theta}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{\theta u_j}+\overline{u_k u_j\theta}-\alpha\overline{u_j\frac{\partial\theta}{\partial x_k}}-\nu\overline{\theta\frac{\partial u_j}{\partial x_k}}\right]+\frac{\partial}{\partial x_j}\overline{p\theta}+\epsilon_{jkl}f_k\overline{u_l\theta}$$
$$=-\overline{u_j u_k}\frac{\partial\Theta}{\partial x_k}-\overline{\theta u_k}\frac{\partial U_j}{\partial x_k}-\beta g_j\overline{\theta^2}+\overline{p\frac{\partial\theta}{\partial x_j}}-(\alpha+\nu)\overline{\frac{\partial u_j}{\partial x_k}\frac{\partial\theta}{\partial k_k}}. \quad (8)$$

$$\frac{\partial\overline{\theta^2}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{\theta^2}+\overline{u_k\theta^2}-\alpha\frac{\partial\overline{\theta^2}}{\partial x_k}\right]$$
$$=-2\overline{u_k\theta}\frac{\partial\Theta}{\partial x_k}-2\alpha\overline{\frac{\partial\theta}{\partial x_k}\frac{\partial\theta}{\partial x_k}}. \quad (9)$$

- To obtain closure, Mellor and Yamada use their own version of the Rotta-Kolmogorov model to approximate higher-moment terms

# Governing Equations

$$\frac{\partial\overline{u_iu_j}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{u_iu_j}+\overline{u_ku_iu_j}-\nu\frac{\partial}{\partial x_k}\overline{u_iu_j}\right]+\frac{\partial}{\partial x_j}\overline{pu_i}$$
$$+\frac{\partial}{\partial x_i}\overline{pu_j}+f_k(\epsilon_{jkl}\overline{u_lu_i}+\epsilon_{ikl}\overline{u_lu_j})$$
$$=-\overline{u_ku_i}\frac{\partial U_j}{\partial x_k}-\overline{u_ku_j}\frac{\partial U_i}{\partial x_k}-\beta(g_j\overline{u_i\theta}+g_i\overline{u_j\theta})$$
$$+\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}-2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}, \quad (7)$$

$$\frac{\partial\overline{u_j\theta}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{\theta u_j}+\overline{u_ku_j\theta}-\alpha\overline{u_j\frac{\partial\theta}{\partial x_k}}-\nu\overline{\theta\frac{\partial u_j}{\partial x_k}}\right]$$
$$+\frac{\partial}{\partial x_j}\overline{p\theta}+\epsilon_{jkl}f_k\overline{u_l\theta}$$
$$=-\overline{u_ju_k}\frac{\partial\Theta}{\partial x_k}-\overline{\theta u_k}\frac{\partial U_j}{\partial x_k}-\beta g_j\overline{\theta^2}+\overline{p\frac{\partial\theta}{\partial x_j}}$$
$$-(\alpha+\nu)\overline{\frac{\partial u_j}{\partial x_k}\frac{\partial\theta}{\partial k_k}}. \quad (8)$$

$$\frac{\partial\overline{\theta^2}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{\theta^2}+\overline{u_k\theta^2}-\alpha\frac{\partial\overline{\theta^2}}{\partial x_k}\right]$$
$$=-2\overline{u_k\theta}\frac{\partial\Theta}{\partial x_k}-2\alpha\overline{\frac{\partial\theta}{\partial x_k}\frac{\partial\theta}{\partial x_k}}. \quad (9)$$

- Energy redistribution hypothesis of Rotta (1951)
  - Suggested could be made proportional to Reynolds stress and mean wind shear

$$p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$
$$=-\frac{q}{3l_1}\left(\overline{u_iu_j}-\frac{\delta_{ij}}{3}q^2\right)+Cq^2\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right), \quad (10)$$

# Governing Equations

$$\frac{\partial \overline{u_i u_j}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k \overline{u_i u_j}+\overline{u_k u_i u_j}-\nu\frac{\partial}{\partial x_k}\overline{u_i u_j}\right]+\frac{\partial}{\partial x_j}\overline{p u_i}$$

$$+\frac{\partial}{\partial x_i}\overline{p u_j}+f_k(\epsilon_{jkl}\overline{u_l u_i}+\epsilon_{ikl}\overline{u_l u_j})$$

$$=-\overline{u_k u_i}\frac{\partial U_j}{\partial x_k}-\overline{u_k u_j}\frac{\partial U_i}{\partial x_k}-\beta(g_j\overline{u_i\theta}+g_i\overline{u_j\theta})$$

$$+\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}-2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}, \quad (7)$$

$$\frac{\partial \overline{u_j\theta}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{\theta u_j}+\overline{u_k u_j\theta}-\alpha\overline{u_j\frac{\partial\theta}{\partial x_k}}-\nu\overline{\theta\frac{\partial u_j}{\partial x_k}}\right]$$

$$+\frac{\partial}{\partial x_j}\overline{p\theta}+\epsilon_{jkl}f_k\overline{u_l\theta}$$

$$=-\overline{u_j u_k}\frac{\partial\Theta}{\partial x_k}-\overline{\theta u_k}\frac{\partial U_j}{\partial x_k}-\beta g_j\overline{\theta^2}+\overline{p\frac{\partial\theta}{\partial x_j}}$$

$$-(\alpha+\nu)\overline{\frac{\partial u_j}{\partial x_k}\frac{\partial\theta}{\partial k_k}}. \quad (8)$$

$$\frac{\partial\overline{\theta^2}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{\theta^2}+\overline{u_k\theta^2}-\alpha\frac{\partial\overline{\theta^2}}{\partial x_k}\right]$$

$$=-2\overline{u_k\theta}\frac{\partial\Theta}{\partial x_k}-2\alpha\overline{\frac{\partial\theta}{\partial x_k}\frac{\partial\theta}{\partial x_k}}. \quad (9)$$

- Kolmogorov hypothesis of local, small-scale isotropy

$$\overline{p\frac{\partial\theta}{\partial x_j}}=-\frac{q}{3l_2}\overline{u_j\theta}. \quad (11)$$

$$2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}=\frac{2}{3}\frac{q^3}{\Lambda_1}\delta_{ij} \quad (12)$$

$$(\alpha+\nu)\overline{\frac{\partial u_j}{\partial x_k}\frac{\partial\theta}{\partial x_k}}=0. \quad (13)$$

$$2\alpha\overline{\frac{\partial\theta}{\partial x_k}\frac{\partial\theta}{\partial x_k}}=2\frac{q}{\Lambda_2}\overline{\theta^2}. \quad (14)$$

# Governing Equations

$$\frac{\partial \overline{u_i u_j}}{\partial t} + \frac{\partial}{\partial x_k}\left[U_k \overline{u_i u_j} + \overline{u_k u_i u_j} - \nu \frac{\partial}{\partial x_k}\overline{u_i u_j}\right] + \frac{\partial}{\partial x_j}\overline{p u_i} + \frac{\partial}{\partial x_i}\overline{p u_j} + f_k\left(\epsilon_{jkl}\overline{u_l u_i} + \epsilon_{ikl}\overline{u_l u_j}\right)$$

$$= -\overline{u_k u_i}\frac{\partial U_j}{\partial x_k} - \overline{u_k u_j}\frac{\partial U_i}{\partial x_k} - \beta\left(g_j \overline{u_i\theta} + g_i\overline{u_j\theta}\right) + \overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)} - 2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}, \quad (7)$$

$$\frac{\partial \overline{u_j\theta}}{\partial t} + \frac{\partial}{\partial x_k}\left[U_k \overline{\theta u_j} + \overline{u_k u_j \theta} - \alpha \overline{u_j\frac{\partial \theta}{\partial x_k}} - \nu \overline{\theta\frac{\partial u_j}{\partial x_k}}\right] + \frac{\partial}{\partial x_j}\overline{p\theta} + \epsilon_{jkl} f_k \overline{u_l\theta}$$

$$= -\overline{u_j u_k}\frac{\partial \Theta}{\partial x_k} - \overline{\theta u_k}\frac{\partial U_j}{\partial x_k} - \beta g_j \overline{\theta^2} + \overline{p\frac{\partial \theta}{\partial x_j}} - (\alpha+\nu)\overline{\frac{\partial u_j}{\partial x_k}\frac{\partial \theta}{\partial k_k}}. \quad (8)$$

$$\frac{\partial \overline{\theta^2}}{\partial t} + \frac{\partial}{\partial x_k}\left[U_k \overline{\theta^2} + \overline{u_k\theta^2} - \alpha\frac{\partial \overline{\theta^2}}{\partial x_k}\right] = -2\overline{u_k\theta}\frac{\partial \Theta}{\partial x_k} - 2\alpha \overline{\frac{\partial \theta}{\partial x_k}\frac{\partial \theta}{\partial x_k}}. \quad (9)$$

- 3rd order moment turbulent velocity diffusion terms are are scaled to 2nd order gradients

$$\overline{u_k u_i u_j} = -q\lambda_1\left(\frac{\partial \overline{u_i u_j}}{\partial x_k} + \frac{\partial \overline{u_i u_k}}{\partial x_j} + \frac{\partial \overline{u_j u_k}}{\partial x_i}\right). \quad (15)$$

$$\overline{u_k u_j \theta} = -q\lambda_2\left(\frac{\partial \overline{u_k\theta}}{\partial x_j} + \frac{\partial \overline{u_j\theta}}{\partial x_k}\right). \quad (16)$$

$$\overline{u_k\theta^2} = -q\lambda_3\frac{\partial \overline{\theta^2}}{\partial x_k}. \quad (17)$$

# Governing Equations

$$\frac{\partial \overline{u_i u_j}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{u_i u_j}+\overline{u_k u_i u_j}-\nu\frac{\partial}{\partial x_k}\overline{u_i u_j}\right]+\frac{\partial}{\partial x_j}\overline{p u_i}$$
$$+\frac{\partial}{\partial x_i}\overline{p u_j}+f_k(\epsilon_{jkl}\overline{u_l u_i}+\epsilon_{ikl}\overline{u_l u_j})$$
$$=-\overline{u_k u_i}\frac{\partial U_j}{\partial x_k}-\overline{u_k u_j}\frac{\partial U_i}{\partial x_k}-\beta(g_j\overline{u_i\theta}+g_i\overline{u_j\theta})$$
$$+\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}-2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}, \quad (7)$$

$$\frac{\partial \overline{u_j\theta}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{\theta u_j}+\overline{u_k u_j\theta}-\alpha\overline{u_j\frac{\partial\theta}{\partial x_k}}-\nu\overline{\theta\frac{\partial u_j}{\partial x_k}}\right]$$
$$+\frac{\partial}{\partial x_j}\overline{p\theta}+\epsilon_{jkl}f_k\overline{u_l\theta}$$
$$=-\overline{u_j u_k}\frac{\partial\Theta}{\partial x_k}-\overline{\theta u_k}\frac{\partial U_j}{\partial x_k}-\beta g_j\overline{\theta^2}+\overline{p\frac{\partial\theta}{\partial x_j}}$$
$$-(\alpha+\nu)\overline{\frac{\partial u_j}{\partial x_k}\frac{\partial\theta}{\partial k_k}}. \quad (8)$$

$$\frac{\partial\overline{\theta^2}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{\theta^2}+\overline{u_k\theta^2}-\alpha\frac{\partial\overline{\theta^2}}{\partial x_k}\right]$$
$$=-2\overline{u_k\theta}\frac{\partial\Theta}{\partial x_k}-2\alpha\overline{\frac{\partial\theta}{\partial x_k}\frac{\partial\theta}{\partial x_k}}. \quad (9)$$

- Some additional simplifications
  - Coriolis force assumed negligible
  - Hanjalic and Launder (1972) assume pressure diffusional terms are small
- Insert closure assumptions into the mean, turbulent momentum equations

$$\overline{pu_i}=\overline{p\theta}=0 \qquad (18)$$

# The Level 4 Model

- Includes all terms in TKE evolution
- Used by Deardorff (1973) to model 3D and unsteady flows
- Since not very practical for most flows, can simplify by ordering terms as products of anisotropy parameter, a, which is assumed to be small, and $q^3/\Lambda$

$$\frac{D\overline{u_i u_j}}{Dt} + f_k(\epsilon_{jkl}\overline{u_l u_i} + \epsilon_{ikl}\overline{u_l u_j})$$

$$= \frac{\partial}{\partial x_k}\left[q\lambda_1\left(\frac{\partial\overline{u_i u_j}}{\partial x_k} + \frac{\partial\overline{u_i u_k}}{\partial x_j} + \frac{\partial\overline{u_j u_k}}{\partial x_i}\right) + \nu\frac{\partial\overline{u_i u_j}}{\partial x_k}\right]$$

$$-\overline{u_k u_i}\frac{\partial U_j}{\partial x_k} - \overline{u_k u_j}\frac{\partial U_i}{\partial x_k} - \beta(g_j\overline{u_i\theta} + g_i\overline{u_j\theta})$$

$$-\frac{q}{3l_1}\left(\overline{u_i u_j} - \frac{\delta_{ij}}{3}q^2\right) + Cq^2\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}\frac{q^3}{\Lambda_1}\delta_{ij}, \quad (19)$$

$$\frac{D\overline{u_j\theta}}{Dt} + f_k\epsilon_{jkl}\overline{u_l\theta}$$

$$= \frac{\partial}{\partial x_k}\left[q\lambda_2\left(\frac{\partial\overline{u_j\theta}}{\partial x_k} + \frac{\partial\overline{u_k\theta}}{\partial x_j}\right) + \alpha\overline{u_j\frac{\partial\theta}{\partial x_k}} + \nu\overline{\theta\frac{\partial u_j}{\partial x_k}}\right]$$

$$-\overline{u_j u_k}\frac{\partial\Theta}{\partial x_k} - \overline{\theta u_k}\frac{\partial U_j}{\partial x_k} - \beta g_j\overline{\theta^2} - \frac{q}{3l_2}\overline{u_j\theta}, \quad (20)$$

$$\frac{D\overline{\theta^2}}{Dt} = \frac{\partial}{\partial x_k}\left[q\lambda_3\frac{\partial\overline{\theta^2}}{\partial x_k} + \alpha\frac{\partial\overline{\theta^2}}{\partial x_k}\right] - 2\overline{u_k\theta}\frac{\partial\Theta}{\partial x_k} - 2\frac{q}{\Lambda_2}\overline{\theta^2}. \quad (21)$$

# Level 3 Model

- From Level 4…
  - Assume diffusion and advection terms are equal and $O(Uq^2/L)$
  - Assume $Uq^2/L=aq^3/\Lambda$
  - Neglect $O(a^2)$ terms
    - Neglects time-rate-of-change, advection, and diffusion terms for anisotropic components of turbulence moments
- Level 3 retains prognostic equations only for TKE and $<\theta^2>$

$$\frac{Dq^2}{Dt}-\frac{\partial}{\partial x_k}\left[\frac{5}{3}q\lambda_1\frac{\partial q^2}{\partial x_k}\right]=-2\overline{u_k u_i}\frac{\partial U_i}{\partial x_k}-2\beta g_k\overline{u_k\theta}-2\frac{q^3}{\Lambda_1} \qquad (20)$$

$$\begin{aligned}\overline{u_i u_j}=&\frac{\delta_{ij}}{3}q^2-\frac{3l_1}{q}\left[(\overline{u_k u_i}-C_1q^2\delta_{ki})\frac{\partial U_j}{\partial x_k}\right.\\&\left.+(\overline{u_k u_j}-C_1q^2\delta_{kj})\frac{\partial U_i}{\partial x_k}-\frac{2}{3}\delta_{ij}\overline{u_k u_l}\frac{\partial U_l}{\partial x_k}\right]\\&-3\frac{l_1}{q}\beta(g_j\overline{u_i\theta}+g_i\overline{u_j\theta}-\tfrac{2}{3}\delta_{ij}g_l\overline{u_l\theta})\\&+3\frac{l_1}{q}\frac{\partial}{\partial x_k}\left[\frac{q\lambda_1}{3}\left(\delta_{ik}\frac{\partial q^2}{\partial x_j}+\delta_{jk}\frac{\partial q^2}{\partial x_i}-\tfrac{2}{3}\delta_{ij}\frac{\partial q^2}{\partial x_k}\right)\right]\end{aligned} \qquad (21)$$

$$\frac{D\overline{\theta^2}}{Dt}-\frac{\partial}{\partial x_k}\left[q\lambda_2\frac{\partial\overline{\theta^2}}{\partial x_k}\right]=-2\overline{u_k\theta}\frac{\partial\Theta}{\partial x_k}-2\frac{q}{\Lambda_2}\overline{\theta^2} \qquad (22)$$

$$\overline{u_j\theta}=-3\frac{l_2}{q}\left[\overline{u_j u_k}\frac{\partial\Theta}{\partial x_k}+\overline{\theta u_k}\frac{\partial U_j}{\partial x_k}+\beta g_j\overline{\theta^2}\right]. \qquad (23)$$

# The Level 2.5 Model

- From Level 3, neglect material derivative and diffusion of potential temperature variance (22)
- Level 2.5 retains isotropic components of transient and diffusive turbulent processes
  - Benefits of Level 3 scheme without computational cost
- Can simplify further by using BL approximation
  - Make hydrostatic assumption
  - Horizontal gradients small
  - Horizontal divergence of turbulent fluxes ignored

$$\frac{Dq^2}{Dt} - \frac{\partial}{\partial x_k}\left[\frac{5}{3} q\lambda_1 \frac{\partial q^2}{\partial x_k}\right] = -2\overline{u_k u_i}\frac{\partial U_i}{\partial x_k} - 2\beta g_k \overline{u_k\theta} - 2\frac{q^3}{\Lambda_1} \quad (20)$$

$$\overline{u_i u_j} = \frac{\delta_{ij}}{3}q^2 - \frac{3l_1}{q}\Big[(\overline{u_k u_i} - C_1 q^2\delta_{ki})\frac{\partial U_j}{\partial x_k} + (\overline{u_k u_j} - C_1 q^2 \delta_{kj})\frac{\partial U_i}{\partial x_k} - \tfrac{2}{3}\delta_{ij}\overline{u_k u_l}\frac{\partial U_l}{\partial x_k}\Big]$$

$$-3\frac{l_1}{q}\beta\left(g_j\overline{u_i\theta} + g_i\overline{u_j\theta} - \tfrac{2}{3}\delta_{ij}g_l\overline{u_l\theta}\right)$$

$$+3\frac{l_1}{q}\frac{\partial}{\partial x_k}\left[\frac{q\lambda_1}{3}\left(\delta_{ik}\frac{\partial q^2}{\partial x_j} + \delta_{jk}\frac{\partial q^2}{\partial x_i} - \tfrac{2}{3}\delta_{ij}\frac{\partial q^2}{\partial x_k}\right)\right] \quad (21)$$

$$\frac{D\overline{\theta^2}}{Dt} - \frac{\partial}{\partial x_k}\left[q\lambda_2\frac{\partial\overline{\theta^2}}{\partial x_k}\right] = -2\overline{u_k\theta}\frac{\partial\Theta}{\partial x_k} - 2\frac{q}{\Lambda_2}\overline{\theta^2} \quad (22)$$

$$\overline{u_j\theta} = -3\frac{l_2}{q}\left[\overline{u_j u_k}\frac{\partial\Theta}{\partial x_k} + \overline{\theta u_k}\frac{\partial U_i}{\partial x_k} + \beta g_j\overline{\theta^2}\right]. \quad (23)$$

# Level 2.5 model

- Use 1.5 order closure and introduce dimensionless variables ($G_h$, $G_m$, $S_h$, $S_m$) to approximate flux terms in (20)-(23)
- $S_q$ chosen as 0.2 …
- Solving this system of equations is very straightforward

$$\frac{D}{Dt}\left(\frac{q^2}{2}\right) - \frac{\partial}{\partial z}\left[lqS_q \frac{\partial}{\partial z}\left(\frac{q^2}{2}\right)\right] = P_s + P_b - \varepsilon \quad (24)$$

$$P_s = -\langle wu\rangle \frac{\partial U}{\partial z} - \langle wv\rangle \frac{\partial V}{\partial z} \quad (25a)$$

$$P_b = \beta g \langle w\theta\rangle_v \quad (25b)$$

$$\varepsilon = q^3/\Lambda_1 \quad (25c)$$

$$-\langle uw\rangle = K_M\, \partial U/\partial z \quad (30a)$$

$$-\langle vw\rangle = K_M\, \partial V/\partial z \quad (30b)$$

$$-\langle \theta w\rangle = K_H\, \partial \Theta/\partial z \quad (31)$$

$$K_M = lq\, S_M \quad (32a)$$

$$K_H = lq\, S_H \quad (32b)$$

$$G_M \equiv \frac{l^2}{q^2}\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] \quad (33a)$$

$$G_H \equiv -\frac{l^2}{q^2}\beta g \frac{\partial \Theta_v}{\partial z} \quad (33b)$$

$$S_M[6A_1A_2G_M] + S_H[1 - 3A_2B_2G_H - 12A_1A_2G_H] = A_2 \quad (34)$$

$$S_M[1 + 6A_1^2G_M - 9A_1A_2G_H] - S_H[12A_1^2G_H + 9A_1A_2G_H] = A_1(1 - 3C_1) \quad (35)$$

# Level 2.5 Model

- All length scales proportional to a single length scale and empirical constants
  - RAMS uses Blackadar's (1962) formula with ratio of TKE moments for $l_0$ and $\alpha$=.10 as suggested by MY74
  - RAMS also assigns an upper limit for l in stable conditions according to André et al. (1978) so that scheme applies to full range of atmospheric forcing

$$l_1, l_2 = A_1 l, A_2 l, \qquad (7a,b)$$

$$\Lambda_1, \Lambda_2 = B_1 l, B_2 l. \qquad (7c,d)$$

$$l = \frac{kz}{1 + kz/l_0}, \qquad (71)$$

$$l_0 = \alpha \frac{\int_0^\infty zq\,dz}{\int_0^\infty q\,dz}, \qquad (72)$$

$$l_D = 0.75\left[\bar{e}^{\frac{1}{2}} / (\alpha g \partial\bar{\theta}/\partial z)^{\frac{1}{2}}\right]. \qquad (14c)$$

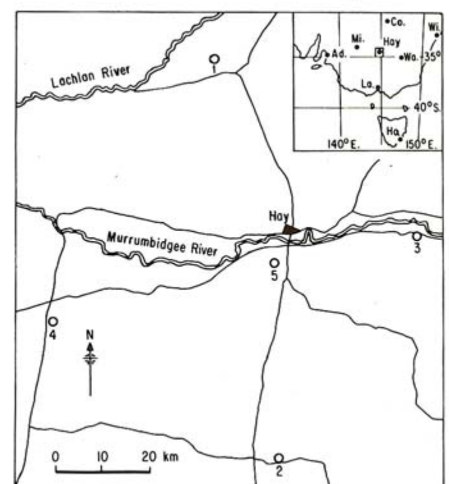
$$l = \min(l, l_D)$$

# Level 2.5 Model

- Empirical constants based on neutral boundary layer and pipe data
- Tested/tuned model against neutral observations from day 33 of Wangara Experiment
  - SE Australia, flat, uniform vegetation, little slope, short sparse grass

$$(A_1, B_1, A_2, B_2, C_1) = (.92, 16.6, 0.74, 10.1, 0.08) \text{ for Mellor-Yamada 82}$$
$$(A_1, B_1, A_2, B_2, C_1) = (.78, 15.0, 0.79, 8.0, 0.23) \quad \text{for Mellor 73}$$

Fig. 1. Photograph of the main site of the Wangara Experiment.



# Level 2.5 Moisture Effects

- Brunt-Väisälä frequency chosen according to moisture levels (as recommended by MY82)

$$N^2 = \begin{cases} g\left[A\dfrac{\partial\theta_e}{\partial z} - \dfrac{\partial q_w}{\partial z}\right] & \text{if moist saturated} \\ g\left[\dfrac{1}{\theta}\dfrac{\partial\theta}{\partial z} + 1.61\dfrac{\partial q_v}{\partial z} - \dfrac{\partial q_w}{\partial z}\right] & \text{if unsaturated} \end{cases}$$

where $A = \theta^{-1}\dfrac{1+\dfrac{1.61\varepsilon L q_v}{R_d T}}{1+\dfrac{\varepsilon L^2 q_v}{C_p R_d T^2}}$, $q_w$ = total water, $q_v$=water vapor, and

$$\theta_e = \theta\left(1+\frac{\varepsilon L q_{vs}}{C_p T}\right),\ q_{vs}\text{=saturation vapor mixing ratio}$$

# Deficiency of Level 2.5

- Designed for the case of near-local equilibrium
  - Performs well for decaying turbulence but fails in growing turbulence because of exclusion of growth rate, advection, vertical diffusion and rapid terms in the balance equations for the second moments

# Changes to Level 2.5

- Level 2.5 has been adapted for case of growing turbulence according to Helfand and Labraga, 1988 (HF88)
  - Isotropy assumption fails when anisotropic terms become too large to ignore and $q^2/q_e^2<1$ (growing convective PBL)
- HL88 recommend the following modification of the nondimensional eddy diffusivities S and velocity variance σ
  - equilibrium values $(\,)_r$ are obtained from level 2 closure which assumes a balance between production and dissipation
  - approximates rapid terms independently from original scheme in terms of known quantities

$$(e)_r = \frac{B_1}{2} l^2 \left[ (S_m)_r \left( \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right) - (S_h)_r N^2 \right]$$

where $(S_h)_r,\ (S_m)_r \ \propto\ R_f,\ R_i$

$$S_m = \sqrt{e/e_r}\,(S_m)_r,\ S_h = \sqrt{e/e_r}\,(S_h)_r$$

$$\sigma_u^2 = e/e_r(\sigma_u^2)_r,\ \sigma_v^2 = e/e_r(\sigma_v^2)_r,\ \sigma_w^2 = e/e_r(\sigma_w^2)_r$$

# RAMSifications of HF88

- Results in continuous but unsmooth transition from growing to decaying turbulence, but the solution is "physically satisfying"
- Produces realistic evolution of the PBL and outperforms several other realizability constraint techniques

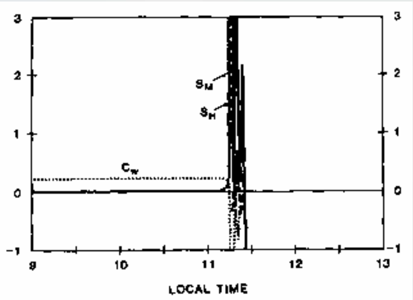
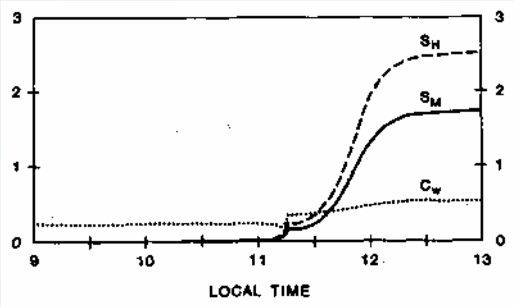


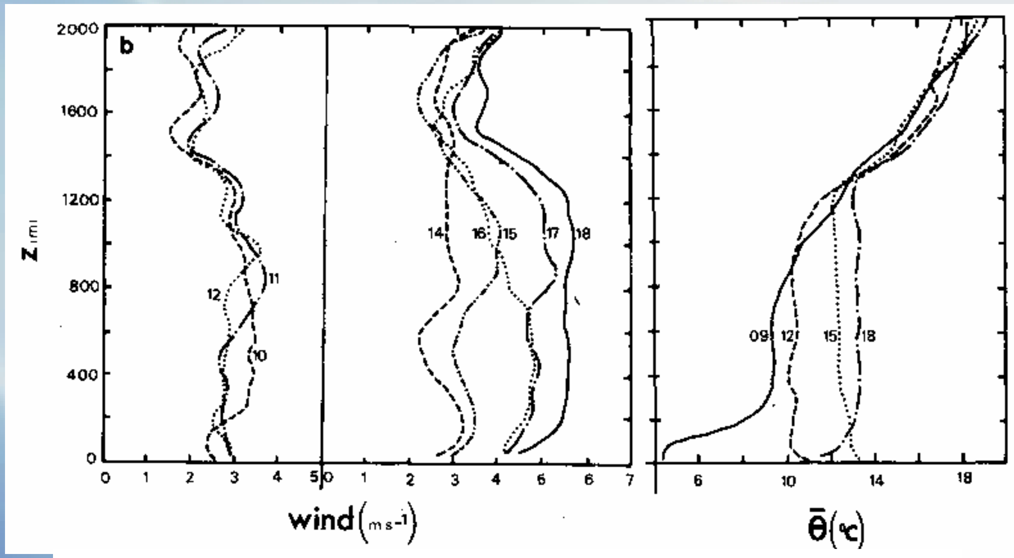

FIG. 10. Time series for the dimensionless coefficients $S_M$, $S_H$, and $C_w$ (solid, dashed and dotted lines, respectively) at the 300 m level for a numerical simulation of the planetary boundary layer with the Mellor–Yamada Level 2.5 model. See text.



# Sensitivity of Model

- Eddy diffusivity is the most important output of the PBL scheme and depends on 8 tunable coefficients
  - $S_e$ , $a_e$, $A_1$, $A_2$, $B_1$, $B_2$, $C_1$, $\alpha$
- Simple test of eddy diffusivity for momentum and heat using neutrally stable BL profile of TKE, potential temperature, and wind from 15 LST, day 33 of Wangara Experiment
- Assess sensitivity to $\alpha$ and to two sets of empirical constants derived by MY82 and M73 using the original level 2.5 scheme

# Boundary Layer Data



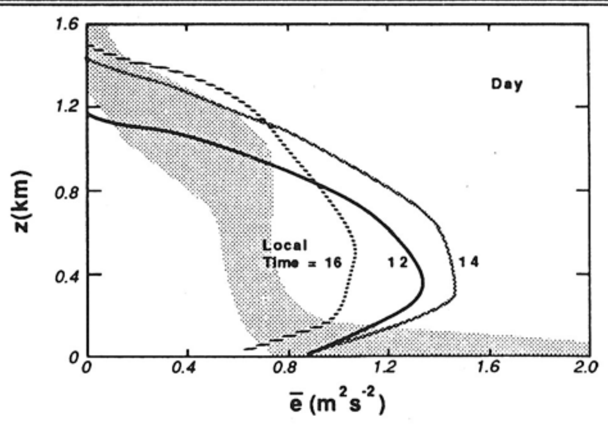
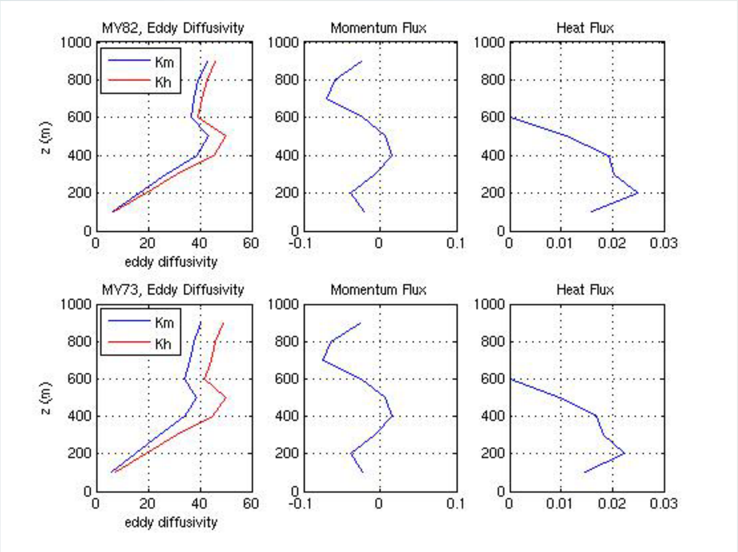
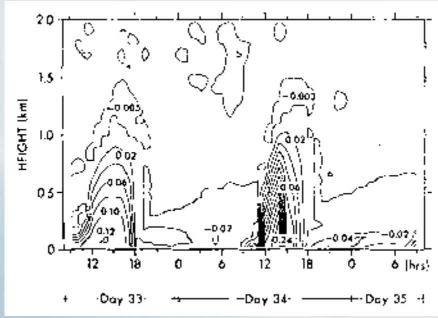



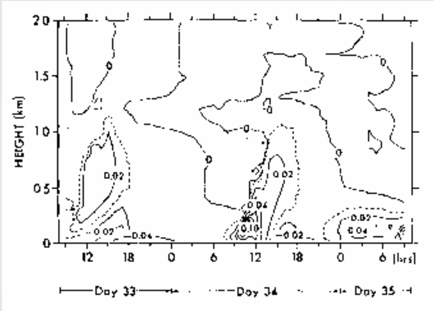

**Fig. 5.3**
The lines show modeled vertical profiles of turbulence kinetic energy, $\overline{e}$, during Day 33, Wangara, (after Therry and Lacarrere, 1983). The shaded profile applies when both shears and buoyancy are active (after Hechtel, 1988).

# Recommended Alpha (.1)

- Eddy diffusivity more spread apart for Mellor 73 values
- Heat flux slightly larger for MY82 values
- Eddy viscosity max is at the correct height but should decrease with height >500m
- Heat and momentum flux compare well to MY model
    - Heat flux is correctly positive below $H_{bl}$ with max at correct height
    - Momentum flux is correctly negative above 500m

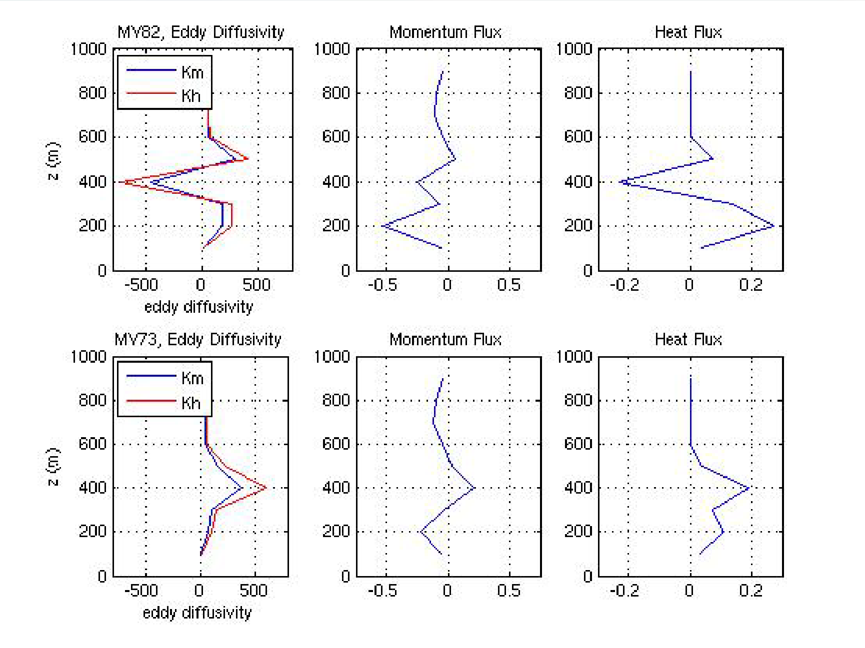


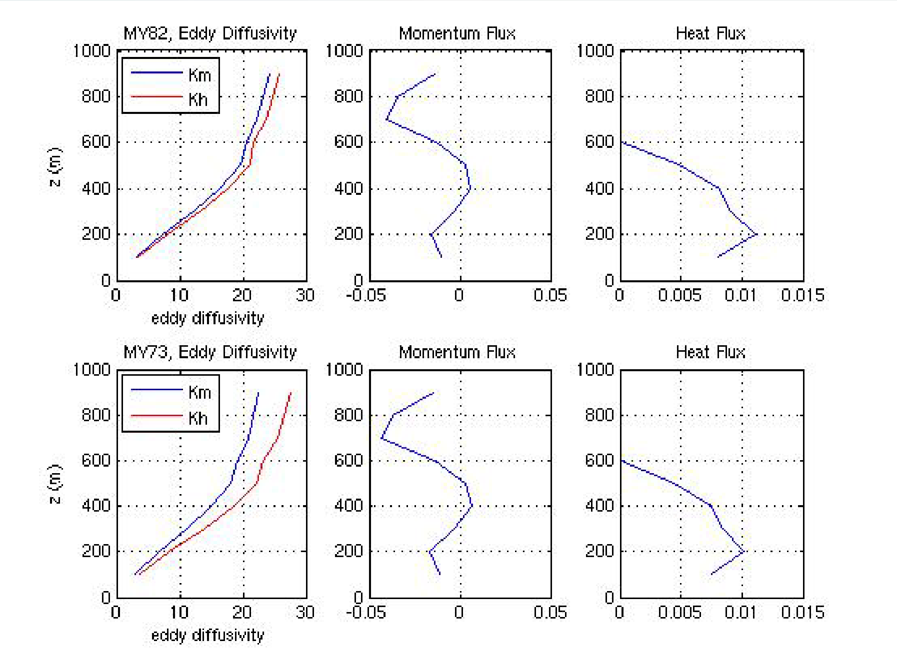
# Doubled Alpha (.2)

- Mixing coefficients blow up in both
  - Shouldn't be larger than ~$|100 m^2 s^{-1}|$
- Momentum and heat flux not as badly affected in MY73 case
- Spikes in heat flux related to spikes in eddy diffusivity



# Half Alpha (.05)

- The response to $\alpha$=.05 is good in that the model doesn't blow up, but the eddy diffusivities are less than half of their original values
- Conclusion: The amount of mixing in the model is fairly strongly dependent on a combination of mixing parameters



# Level 2.5 in RAMS

- Compute vertical wind shear and $N^2$ separately and then feed into PBL subroutine
- Given TKE, wind shear, and $N^2$ for each grid point and vertical level, compute master length scale, nondimensional vertical gradients ($G_h$, $G_m$), nondimensional eddy diffusivities ($S_h$, $S_m$), and eddy diffusivities for heat and momentum for 2 scenarios:
  1. tker > tkep (growing turbulence)
  2. tker <= tkep (neutral/decaying turbulence)
- Excluding external variables, each case requires the calculation of about 10 dependent variables

# Lookup Table

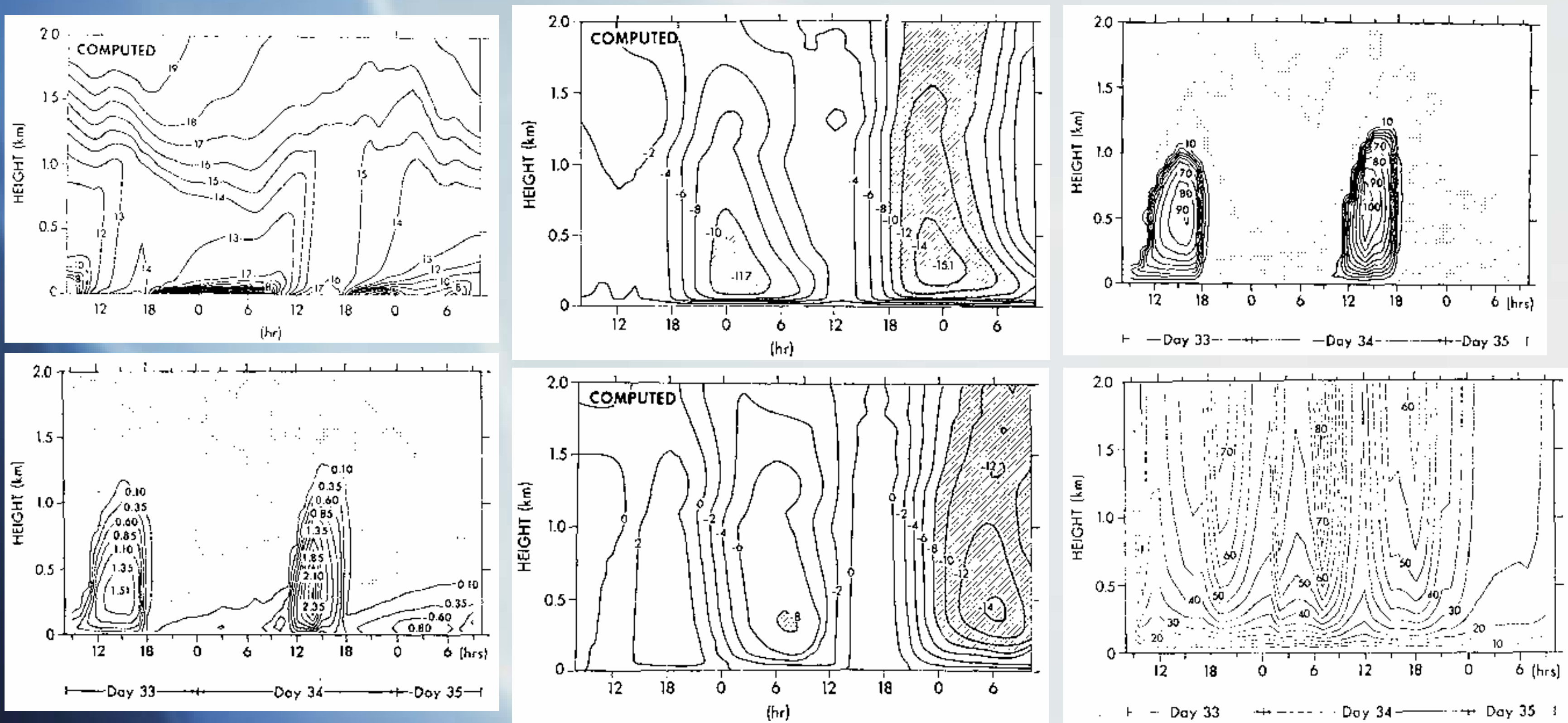
- Matsui et al (2004) notes that the computational burden of parameterizations can be reduced in two different ways
  1. Reduce level of complexity of model
  2. Pre-compute every possible output and store in a LUT
- Since there are only three inputs into the model and only a few equations to solve, it seems that the first approach been implicitly satisfied
- Plots of eddy diffusivity, however, leads me to believe that the range of possible outputs is fairly limited and a LUT could be at least mildly beneficial

# Hybrid LUT

- Allow model to solve for master length scale (l)
- Call LUT after l is computed
  - Input is l, tke, shear, and $N^2$
  - Output is the updated eddy diffusivity and TKE
- Use Similarity Theory and Buckingham Pi Theory to reduce inputs (as in Zinn et al, 1995)
  - 4 variables (m): square of wind shear ($sh = 1/S^2$), Brunt-Vaisalla frequency ($en2 = 1/S^2$), mixing length ($l = L$), tke ($e = L^2/S^2$)
  - 2 dimensions (n): S and L
  - Key variables (m-n=2): en2, e
  - Dimensionless Pi groups: $\pi_1$=en2/sh, $\pi_2$=l*(sh/e)$^{1/2}$

# Hybrid LUT

- Additional Steps:
  - The next step would be to perform an experiment to determine values of the dimensionless groups
  - Next, fit an empirical curve or regress an equation to data to describe relationship between groups
  - Develop equations to relate eddy diffusivity and/or TKE to the dimensionless groups
  - Create a lookup table from the equations and compare results to parameterization results and observations
- Level 3 output indicate that it might be possible to relate eddy diffusivity to $\pi_1$ and $\pi_2$




- 12 LST: neutral/stable, tke growing, little wind shear, l is small --> Km growing
- 3 LST: stable, tke~0, some wind shear, l is average --> Km~0

# References


- Matsui, T., G. Leoncini, R.A. Pielke Sr., and U.S. Nair, 2004: A new paradigm for parameterization in atmospheric models:  Application to the new Fu-Liou radiation code, Atmospheric Science Paper No. 747, Colorado State University, Fort Collins, CO 80523, 32 pp.
- Zinn, H.P., Kowalski, A.D., An efficient PBL Model For Global Circulation Models - Design and Validation, Boundary-Layer Meteorology, 75: 25-59, 1995

# Shortcomings of Model

- Tuned against homogeneous neutral atmosphere and not designed for rapidly growing turbulence
- Turbulent length scale is not clearly defined
- Boundary layer height typically underestimated