Coriolis Force in the WRF model

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Equations

- $U(t+\Delta t) = U(t) + \Delta t * F_{U,Cor}$
- $V(t+\Delta t) = V(t) + \Delta t * F_{V,Cor}$
- $W(t+\Delta t) = W(t) + \Delta t * F_{W,Cor}$

- This is a forward in time method
- The WRF model describes both the horizontal and vertical Coriolis forces

Force Equations

- The forces $F_{U,Cor}$, $F_{V,Cor}$, $F_{W,Cor}$ are described by the following equations:
- $F_{U,Cor} = [(f_{i+1/2} + f_{i-1/2})/2]*[(V_{i+1/2,j+1/2} + V_{i+1/2,j-1/2} + V_{i-1/2,j+1/2} + V_{i-1/2,j-1/2})/4] – [(e_{i+1/2} + e_{i-1/2})/2]*[W_{i+1/2,k+1/2} + W_{i+1/2,k-1/2} + W_{i-1/2,k+1/2} + W_{i-1/2,k-1/2})/4]*[(\cos\alpha_{i+1/2} + \cos\alpha_{i-1/2})/2]$
- $F_{V,Cor} = -[(f_{j+1/2} + f_{j-1/2})/2]*[(U_{i+1/2,j+1/2} + U_{i+1/2,j-1/2} + U_{i-1/2,j+1/2} + U_{i-1/2,j-1/2})/4] – [(e_{j+1/2} + e_{j-1/2})/2]*[W_{j+1/2,k+1/2} + W_{j+1/2,k-1/2} + W_{j-1/2,k+1/2} + W_{j-1/2,k-1/2})/4]*[(\sin\alpha_{j+1/2} + \sin\alpha_{j-1/2})/2]$
- $F_{W,Cor} = e*\{[(U_{i+1/2,k+1/2} + U_{i+1/2,k-1/2} + U_{i-1/2,k+1/2} + U_{i-1/2,k-1/2})/4]*\cos\alpha - [(V_{j+1/2,k+1/2} + V_{j+1/2,k-1/2} + V_{j-1/2,k+1/2} + V_{j-1/2,k-1/2})/4]*\sin\alpha\}$

Force Equations

- These equations reduce to the following when we remove the grid staggering:

- $F_{U,Cor} = f*V - e*W*\cos\alpha$
- $F_{V,Cor} = -f *U - e *W *\sin \alpha$
- $F_{W,Cor} = e*U *\cos \alpha - V *\sin \alpha$

Force Equations

- This leaves us with the equations for the Coriolis Force as:

- $U(t+\Delta t) = U(t) + \Delta t^{*}\ (f^{*}V - e^{*}W^{*}\cos\alpha)$
- $V(t+\Delta t) = V(t) + \Delta t^{*}(-f\ {}^{*}U - e\ {}^{*}W\ {}^{*}\sin\alpha)$
- $W(t+\Delta t) = W(t) + \Delta t^{*}(e^{*}U\ {}^{*}\cos\alpha - V\ {}^{*}\sin\alpha)$

Force Equations

- where α is the local rotation angle between the y-axis and the meridians
- $e = 2\Omega\cos\varphi$
- $f = 2\Omega\sin\varphi$
- φ is the latitude
- Includes both horizontal and vertical effects

Stability Analysis

- Let $u = u_o \exp[i(kn\Delta x + \omega\tau\Delta t)]$
- $v = v_o \exp[i(kn\Delta x + \omega\tau\Delta t)]$
- $w = w_o \exp[i(kn\Delta x + \omega\tau\Delta t)]$
- and $\Psi = \exp(i\omega\Delta t)$. Plugging these values into the equations and putting it in matrix form we get:

Stability Analysis

$$\begin{vmatrix} \Psi-1 & -f\,\Delta t & e\cos\alpha\,\Delta t \\ f\,\Delta t & \Psi-1 & -e\sin\alpha\,\Delta t \\ -e\cos\alpha\,\Delta t & e\sin\alpha\,\Delta t & \Psi-1 \end{vmatrix} \begin{vmatrix} u_o \\ v_o \\ w_o \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

- We need to find the determinant of the matrix and set it equal to zero

Stability Analysis

- When we set the determinant equal to zero we get

$$(\Psi\text{-}1)[(\Psi\text{-}1)^2 + e^2\Delta t^2 + f^2\Delta t^2] = 0$$

- Now we need to solve for Ψ

Stability Analysis

- Solving for we get $\Psi = 1,\ 1 \pm i\,\Delta t\,\sqrt{(e^2+f^2)}$
- Equating real and imaginary parts we get:

  $\lambda\cos(\omega_r\Delta t) = 1$

  $\lambda\sin(\omega_r\Delta t) = \pm\Delta t\sqrt{(e^2+f^2)}$
- Squaring and summing we find that

  $\lambda^2 = 1 + \Delta t^2(e^2+f^2)$

Stability Analysis

- Solving for λ we find that
  $\lambda = \sqrt{[1+\Delta t^2(e^2+f^2)]} \geq 1$
- This shows that $\lambda = 1$ only when $\Delta t = 0$
- The scheme for the Coriolis force is unstable in WRF

References

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