Vertical Subgrid Scale Mixing in WRF

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Overview

- Stability analysis of sub-grid scale mixing
- Review options for explicit vertical mixing in WRF
- Evaluate PBL schemes

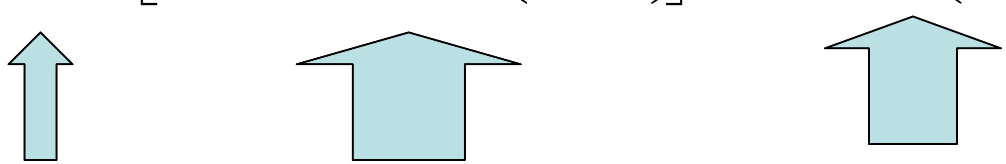
Intro to Vertical Mixing in WRF

- Explicit Spatial Diffusion
  - Used when most boundary layer eddies can be resolved by the dynamics of the model ($\Delta x, \Delta y \sim \Delta z$)
  - Assume fully three-dimensional local sub-grid turbulence
- PBL Parameterization
  - Clear scale separation between sub-grid eddies and resolvable eddies ($\Delta x, \Delta y > \Delta z$)
  - Full physics Numerical Weather Prediction mode
  - Local and non-local closure schemes

Explicit Vertical Mixing

- The goal of explicit schemes is to estimate the vertical eddy viscosity (K_v) for the diffusion terms of the the dynamical equations (the momentum equation for example), without additional physics to compromise computational efficiency

$$\partial_t(\mu_d a) = \ldots + \mu_d \left[m\partial_x (mK_h \partial_x a) + m\partial_y (mK_h \partial_y a) \right] + g^2 (\mu_d \alpha)^{-1} \partial_\eta (K_v \alpha^{-1} \partial_\eta a)$$



Model Physics + Dynamics

Horizontal Diffusion =

$$\frac{\partial}{\partial x}\left(\overline{u_i' a'}\right) + \frac{\partial}{\partial y}\left(\overline{u_i' a'}\right)$$

Vertical Diffusion =

$$\frac{\partial}{\partial \eta}\left(\overline{u_i' a'}\right)$$

Numerical Stability Analysis

- Linearize prognostic equation for model variable ‘a’ with the following assumptions until left with a simplified diffusion equation:
  - One-dimensional (vertical)
  - Kv, μ, α constant in space and time
  - Let $C = K_v g^2 \alpha^{-2}$ = constant

$$\partial_t(\mu_d a) = \ldots + \mu_d\left[m\partial_x(mK_h\partial_x a) + m\partial_y(mK_h\partial_y a)\right] + g^2(\mu_d\alpha)^{-1}\partial_\eta(K_v\alpha^{-1}\partial_\eta a)$$

$$\frac{\partial a}{\partial t} = C\frac{\partial^2 a}{\partial \eta^2}$$

- Use a finite difference scheme that is forward in time, centered in space. This is natural for diffusion since it is a symmetrical process.

$$\frac{w_{i,j,z}^{n+1} - w_{i,j,z}^{n}}{\Delta t} = C\frac{w_{i,j,z+1}^{n} - 2w_{i,j,z}^{n} + w_{i,j,z-1}^{n}}{(\Delta \eta)^2}$$

- Apply von Neumann's method.

$$\hat{w}^{n+1} = \lambda \hat{w}^{n}, \text{ where } \lambda \text{ is the amplification factor}$$

- Rewrite finite difference equation, require that abs(λ) ≤1 for stability, let C absorb Δt / ($\Delta\eta$)^2 for simplicity.

$$\lambda w_{i,j,z}^{n} = w_{i,j,z}^{n} + C\left(w_{i,j,z+1}^{n} - 2w_{i,j,z}^{n} + w_{i,j,z-1}^{n}\right)$$

- Look for wavelike solution to w.

$$w_{z}^{n} = \hat{w}^{n} e^{ikz\Delta\eta}$$

$$w_{z+1}^{n} = \hat{w}^{n} e^{ik(z+1)\Delta\eta}$$

$$w_{z-1}^{n} = \hat{w}^{n} e^{ik(z-1)\Delta\eta}$$

- Substitute wave solutions into finite difference equation, cancel terms.

$$\lambda - 1 = C\left(e^{ik\Delta\eta} - 2 + e^{-ik\Delta\eta}\right)$$

- Simplify using Eulers method.

$$\lambda - 1 = 2C\left(\cos\left(k\Delta\eta\right) - 1\right)$$

- Apply trigonometric identity.

$$\lambda = 1 - 4C\sin^2\left(\frac{k\Delta\eta}{2}\right)$$

- Instability can occur if λ<-1.

$$\lambda = 1 - 4C\sin^2\left(\frac{k\Delta\eta}{2}\right) < -1$$

$$= C\sin^2\left(\frac{k\Delta\eta}{2}\right) > \frac{1}{2} \quad \text{or} \quad C\sin^2\left(\frac{k\Delta\eta}{2}\right) \le \frac{1}{2} \ \text{for stability}$$

-The worst case scenario is $\sin^2(k\Delta\eta/2)=1$, which occurs for the shortest resolvable wave $L=2\Delta\eta=2\pi/k$.

$$\lambda = C\sin^2\left(\frac{k\Delta\eta}{2}\right) \leq \frac{1}{2}$$

$$= C\sin^2\left(\frac{\pi}{2}\right) \leq \frac{1}{2}$$

$$= C \leq \frac{1}{2} \text{ or } \Delta t \leq \frac{(\Delta\eta)^2}{2K_v} \text{ is the minimum requirement for stability}$$

- We conclude that the time step is proportional to square of vertical grid spacing and inversely proportional to eddy diffusivity. So if the grid spacing is reduced by half then we must equivalently reduce the time step by one quarter to maintain the minimum level of stability.

Options for Explicit Vertical Mixing

- 3D Smagorinsky Closure
- Prognostic TKE Closure
- Both of these schemes assume local mixing only

3D Smagorinsky Closure

- Relates mixing coefficients to the fluid deformation rate
- The equation for eddy diffusivity is based on properties of flow
- Several authors have demonstrated sensitivity of simulated squall lines to C_s

$$K_{h,v} = C_s^2 l_{h,vf}^2 \max\left[0., D^2 - \left(\Pr^{-1} N^2\right)^{1/2} \right]$$

SGS Constant

Length scale of largest eddies

Reynolds Stress (Shear)

Buoyancy, includes moisture effects

Example of sensitivity to C_s from Takema and Rotunno (2003)

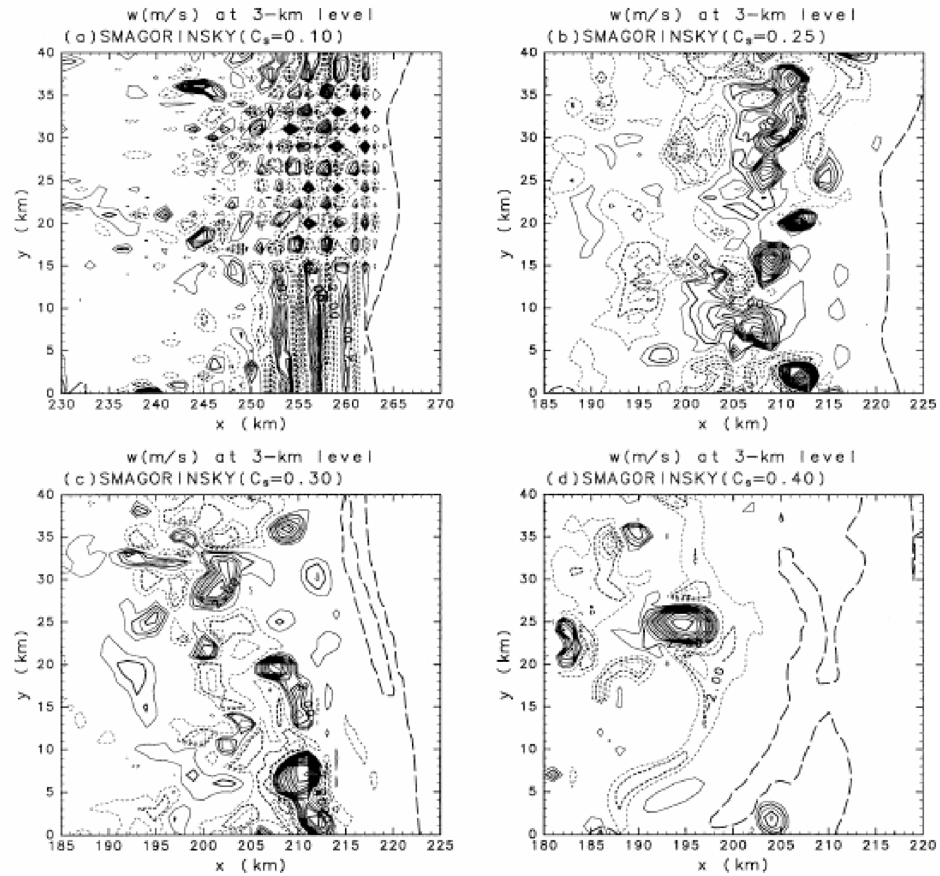


FIG. 4. Same as in Fig. 1 except for the Smagorinsky scheme with (a) C_s = 0.1, (b) C_s = 0.25, (c) C_s = 0.3, and (d) C_s = 0.4.

Prognostic TKE Closure

- Solve prognostic equation for TKE and then plug solution into equation for eddy viscosity
  - Has the advantage of capturing the physical processes that govern the evolution of turbulence throughout the boundary layer for most atmospheric conditions
  - Includes a budget for transient and diffusive effects of turbulence, and production and dissipation of turbulence
- Designed for high-resolution cloud-resolving and eddy-resolving simulations

$$K_{h,v} = C_k l_{h,v} \sqrt{e}, \text{ where } e = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$$

$l_{h,v} \equiv$ master length scale $\propto \Delta z$ or $\dfrac{\sqrt{e}}{N}$ depending on stability and grid spacing

$C_k \rightarrow$ controls amount of physical diffusion,
optimal value reduces need for numerical filter,
insufficiently high value yields poorly resolved grid-scale features (noise)

TKE Formulation

$$\partial_t(\mu_d e) + (\nabla \bullet Ve)_\eta = \mu_d(\text{shear} + \text{buoyancy} + \text{dissipation})$$

$$\text{shear} = \mathrm{K_h} D_{11}^2 + \mathrm{K_h} D_{22}^2 + \mathrm{K_v} D_{33}^2 + \overline{\mathrm{K_h} D_{12}^2}^{xy} + \overline{\mathrm{K_v} D_{13}^2}^{x\eta} + \overline{\mathrm{K_v} D_{23}^2}^{y\eta}$$

$$\propto K\left[\left(\frac{\partial u_i}{\partial x_j}\right)^2 + \left(\frac{\partial u_j}{\partial x_i}\right)^2\right]$$

$$\text{buoyancy} = -K_v N^2,\ N^2 = \begin{cases} g\left(A\dfrac{\partial \theta_e}{\partial z} - \dfrac{\partial q_w}{\partial z}\right) & \text{for moist saturated env} \\ g\left(\dfrac{1}{\theta}\dfrac{\partial \theta}{\partial z} + 1.61\dfrac{\partial q_v}{\partial z} - \dfrac{\partial q_w}{\partial z}\right) & \text{for unsaturated env} \end{cases}$$

$$\text{dissipation} \propto -\frac{e^{3/2}}{l},\ l = \begin{cases} \min\left[(\Delta x \Delta y \Delta z)^{1/3}, 0.76\sqrt{e}/N\right] & \text{for } \Delta\text{x} < l_{cr} \\ \dfrac{kz}{l + kz/l_o} & \text{for } \Delta\text{x} > l_{cr} \end{cases}$$

PBL Schemes

- Medium Range Forecast Model (MRF) PBL
- Yonsei University (YSU) PBL
- Mellor-Yamada-Janjic (MYJ) PBL

MRF PBL

- Based on Troen and Mahrt (1984) non-local K scheme
- Rapid non-local mixing transports heat/moisture away from surface
  - Better representation of large scale precipitation but not convective precipitation
- Represents mixed layer and free atmosphere separately
- Treats entrainment as part of PBL mixing, estimates PBL top from bulk Richardson number and virtual potential temperature

- Turbulent diffusion has terms for local and non-local mixing.

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial z}\left[K_c\left(\frac{\partial c}{\partial z} - \gamma_c\right)\right]$$

$$\text{where } K_{m,t} = \begin{cases} k w_s z\left(1-\frac{z}{h}\right) & \text{for mixed layer} \\ l^2 f_{m,t}(Rig)\left|\frac{\partial u}{\partial z}\right| & \text{for free atmosphere} \end{cases}$$

$$\gamma_c = b\frac{\overline{w'c'}}{w_x} \equiv \text{counter-gradient correction term (0 in free atmosphere)}$$

-Eddy diffusivity for heat ($K_{z,t}$) obtained from Prandtl number and eddy diffusivity for momentum ($K_{m,t}$)

- Eddy diffusivity in the boundary layer is based on an idealized profile that is continuous with height and consistent with values at the top of the surface layer.

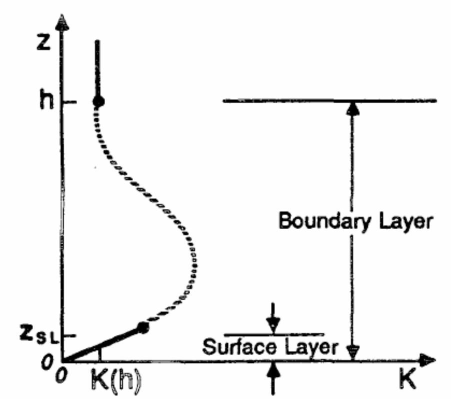


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

YSU PBL

- Next generation MRF
- Explicit representation of entrainment derived from large eddy simulation
- Alleviates problem of excessive entrainment during early PBL growth
- Adds non-local momentum mixing for more realistic wind profile in PBL
- Removes influence of convective velocity on surface stress, which increases daytime low-windspeed bias

MYJ PBL

- Original level 2.5 scheme for neutral/stable BL with modifications for full range of turbulence
  - Upper limit on length scale $\propto$ TKE/buoyancy/shear
  - Require non-singular TKE production for growing turbulence
  - $(w'2/TKE) > (w'2/TKE)_{\text{vanishing turb}}$
  - New empirical constants that do better job relating Kolmogorov and Rotta length scales to the master length scale for unstable boundary layer
- 1.5 order local K scheme for entire atmospheric column
  - Similar formulation as prognostic TKE scheme for explicit diffusion except designed for large grid spacing and has more sophisticated formulation for proportionality constant of eddy diffusivity equation

Summary of PBL Schemes

Table 8.4: Planetary Boundary Layer Options

Scheme	Unstable PBL Mixing	Entrainment treatment	PBL Top
MRF	K profile + countergradient term	part of PBL mixing	from critical bulk Ri
YSU	K profile + countergradient term	explicit term	from buoyancy profile
MYJ	K from prognostic TKE	part of PBL mixing	from TKE