Integrating models and observations
Data assimilation / Inverse modeling

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Relationship between model and observations

Forward problem

\[ y = f(m) \]

\( f \) - model
\( m \) - input that model result depends on
\( y \) - simulated observations

Is \( y \) “matching” actual observations?

Inverse problem (data assimilation)

Find \( m \) such that \( y \) “matches” actual observations
Probability means: probability of occurrence of a value

\[ p_m(m) \approx \int_D \left[ p_2(m) p_2(y) p_1(y/m) \right] dy \]

- Probability of \( m \) after - posterior
- Probability of \( m \) before - prior
- Probability of observations
- Includes how much is known about \( m \) before using observations
- Includes observation values and observation errors
- Probability of modeled observations
- Includes model result and model errors
- Integrated over all observations
Examples to illustrate not to prove inverse theory

• The theory is already proven

• In practical data assimilation (examples from O’Neill) the theory is applied approximately, within practical limits: resolution, data volume etc
Example 1: **Damped oscillations model**

Observation is oscillation amplitude

\[ \chi = A_1 e^{(-\lambda + \eta) \tau} + A_2 e^{(-\lambda - \eta) \tau} \]

**m is initial condition**

**m is natural frequency**

\[ p_m(m) \approx \int_D \left[ p_2(m) p_2(y) p_1(y/m) \right] dy \]
Example 2: **Lorenz 3-component model**

Observations in X component at different times

Dots represent min and max values in the observation range for each observation time.

Observations every 10 time steps.
m is vector of model coefficients

Example of 2 different observation times

Complex $p(m)$ for each observation individually
Inverse result for all parameters in the model and all observations combined

Green is true value
Red is final result of assimilation

No model error
Moderate error
Large error
Possible structural model error
Summary

- Excellent results without model error
- Good results with non-structural model error
- Non-informative results with large model error

Could models be improved by data assimilation?

- New research
  - climate and weather/process models