# Short-term prediction of measurement precision of the radial wind velocity in the planetary boundary layer

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# Doppler shift estimate

Doppler shift estimate:

$$\hat{f}_d = \frac{1}{\mathbf{S}_0} \sum_m \omega_m \hat{\mathbf{S}}(\omega_m)$$

Estimate of spectrum:

$$\hat{\mathbf{S}}(\omega_m) = \frac{1}{M_p} \sum_{j=1}^{M_p} T_s^2 \left| \sum_{n=0}^{M_s-1} j(t_j + nT_s) e^{-i\omega_m n T_s} \right|^2$$

$$\mathbf{S}_0 = \sum_{m=0}^{M_s-1} \hat{\mathbf{S}}(\omega_m)$$

Doppler lidar signal:

$$j(t) = j_s(t) + j_n(t)$$

Aerosol signal:

$$j_s(t) = \sum_{m=1}^{N_p} A_m P(t, \mathbf{r}_m) e^{2ikz_m^d(t)}$$

Noise:

$$K_n(t_2, t_1) = \left\langle j_n^*(t_2) j_n(t_1) \right\rangle = \mathrm{N}\delta_{t_1, t_2}$$

where $M_s$ is number of samples, $T_s$ is time of samples,

$M_p$ is number of averaged pulses, $j(t)$ is Doppler lidar signal.

# Doppler shift estimate

$$\hat{f}_d(t,z) = f(t,z) + f_g(t,z) + f_{ng}(t,z)$$

$$f(t,z) = 2k\left(U(t,z)\mathbf{i}_1 + V(t,z)\mathbf{i}_2\right)\mathbf{n}$$ **is regular component,**

$$f_{ng}' = \frac{2k}{M_s}\sum_{q=0}^{M_s-1}\int\left|p\left(qT_s - 2\frac{|L-z_1|}{c}, \mathbf{r}_1\right)\right|^2 u_r'(\mathbf{r}_1)d\mathbf{r}_1$$ **is non-Gaussian fluctuating component,**

$$f_g' = \sum_m \left(\omega_m - f_{ng}'\right)\Delta\hat{\mathbf{S}}(\omega_m)/\mathbf{S}_0$$ **is Gaussian fluctuating component.**

# Equation for estimation of the radial wind velocity

**Estimation of radial wind velocity:**

$$\hat{u}_r(t,z) = u_r(t,z) + \Delta u_r(t,z)$$

**Estimation of radial wind velocity:** $$\hat{u}_r(t,z) = \frac{1}{2k}\hat{f}_d(t,z)$$

**Average radial wind velocity:** $$u_r(t,z) = \left(U(t,z)\mathbf{i}_1 + V(t,z)\mathbf{i}_2\right)\mathbf{n}$$

**Random error :** $$\Delta u_r(t,z) = \frac{1}{2k}\left[f_{ng}'(t,z) + f_g'(t,z)\right]$$

# Standard Deviation of random error

1. $\varepsilon(t,z)d_{\mathrm{v,eff}} << (e(t,z))^{3/2}$

$$\mathrm{StdDiv}(\mathrm{m}^2/\mathrm{s}^2) = \frac{1}{3}e(t,z) - 0.401C^2\left(\varepsilon(t,z)d_{\mathrm{v,eff}}\right)^{2/3} + \frac{1}{8k^2M_pM_sT_s^2}\left\{2kT_s\sqrt{\pi\left(\frac{1}{8k^2\tau_0^2}+0.401C^2(\varepsilon(t,z)d_{\mathrm{v}})^{2/3}\right)}\right. +$$

$$\left. +2\frac{\mathrm{N}}{\mathrm{S}}\left[1-\left(1-8k^2T_s^{\ 2}\left(\frac{1}{8k^2\tau_0^2}+0.401C^2(\varepsilon(t,z)d_{\mathrm{v}}\right)\delta_{T,T_1}+\frac{\mathrm{N}^2}{\mathrm{S}^2}\right)\right]\right.$$

# Standard Deviation of random error

$$2.\ \varepsilon(t,z)d_{\mathrm{v,eff}} >> (e(t,z))^{3/2} >> \varepsilon(t,z)d_{\mathrm{v}}$$

$$\mathrm{StdDiv}\,(\mathrm{m}^2/\mathrm{s}^2) = \frac{1}{8k^2 M_p M_s T_s^2}\left\{2kT_s\sqrt{\pi\left(\frac{1}{8k^2\tau_0^2} + 0.401C^2(\varepsilon(t,z)d_{\mathrm{v}})^{2/3}\right)} + \right.$$

$$\left. + 2\frac{\mathrm{N}}{\mathrm{S}}\left[1 - \left(1 - 8k^2 T_s^{\,2}\left(\frac{1}{8k^2\tau_0^2} + 0.401C^2(\varepsilon(t,z)d_{\mathrm{v}}\right)\delta_{T,T_1} + \frac{\mathrm{N}^2}{\mathrm{S}^2}\right)\right]\right.$$

# Standard Deviation of random error

3. $\varepsilon(t,z)d_{\mathrm{v}} >> \left(e(t,z)\right)^{3/2}$

$$\mathrm{StdDiv}\ (\mathrm{m}^2/\mathrm{s}^2) = \frac{1}{8k^2 M_p M_s T_s^2}\left\{2kT_s\sqrt{\pi\left(\frac{1}{8k^2\tau_0^2}+e(t,z)\right)}+\right.$$
$$\left.+2\frac{\mathrm{N}}{\mathrm{S}}\left[1-\left(1-8k^2T_s^{\ 2}\left(\frac{1}{8k^2\tau_0^2}+e(t,z)\right)\right)\delta_{T,T_1}+\frac{\mathrm{N}^2}{\mathrm{S}^2}\right]\right\}$$

# “e-$\varepsilon$”-turbulence model

$$\frac{\partial e}{\partial t} = -\langle u'w'\rangle \frac{\partial U}{\partial z} - \langle v'w'\rangle \frac{\partial V}{\partial z} + \frac{Q}{\Theta}\langle \theta' w'\rangle + \frac{\partial}{\partial z}\left(\sigma_e \sqrt{el}\,\frac{\partial e}{\partial z}\right) - \frac{C_D e^{3/2}}{l}$$

$$\frac{\partial l}{\partial t} = C_{L1}\left(-\langle u'w'\rangle \frac{\partial U}{\partial z} - \langle v'w'\rangle \frac{\partial V}{\partial z} + \frac{g}{\Theta}\langle \theta' w'\rangle\right)\frac{l}{e} + \frac{\partial}{\partial z}\left(\sigma_e \sqrt{el}\,\frac{\partial l}{\partial z}\right) + C_{L2}\sqrt{e}\left[1-\left(\frac{1}{kz}\right)^3\right]$$

$$\left\{\langle u'w'\rangle, \langle v'w'\rangle, \langle \theta' w'\rangle\right\} = -F_{m,h}\sqrt{el}\left\{\frac{\partial U}{\partial z}, \frac{\partial V}{\partial z}, \frac{\partial \Theta}{\partial z}\right\}$$

$$\varepsilon = \frac{C_D e^{3/2}}{l}$$

where $e(t,z)$ is kinetic energy, $\varepsilon(t,z)$ is energy dissipation rate, $l(t,z)$ is scale of turbulence. The “$e$-$\varepsilon$” turbulence model used corresponds to the 2.5 level by the Mellor-Yamada classification.

**A.V. Starchenko and D.A. Belikov**
“A numerical model for real-time monitoring of urban air quality”. Atmos. Oceanic Opt. July 2003. Vol. 16, No. 7, pp. 608-615.

# 1D model of planetary boundary layer

$$\frac{\partial U}{\partial t} = -\frac{\partial}{\partial z}\langle u'w'\rangle + f\left(V - V_g\right)$$

$$\frac{\partial V}{\partial t} = -\frac{\partial}{\partial z}\langle v'w'\rangle + f\left(U - U_g\right)$$

$$\frac{\partial \Theta}{\partial t} = -\frac{\partial}{\partial z}\langle \theta'w'\rangle$$

$$\frac{\partial Q}{\partial t} = -\frac{\partial}{\partial z}\langle q'w'\rangle$$

where U and V are components of the mean horizontal wind velocity, $\Theta$ is potential temperature, Q is humidity.

$$\left\{V_g, U_g\right\} = \frac{1}{\rho f}\left\{\frac{\partial p}{\partial y}, \frac{\partial p}{\partial x}\right\}$$ are the components of the geostrophic wind.

# The boundary conditions

- $z = z_1 >> z_0$

$$U = \frac{v_*}{\kappa} f_u(\xi_1)\cos\beta \qquad V = \frac{v_*}{\kappa} f_u(\xi_1)\sin\beta$$

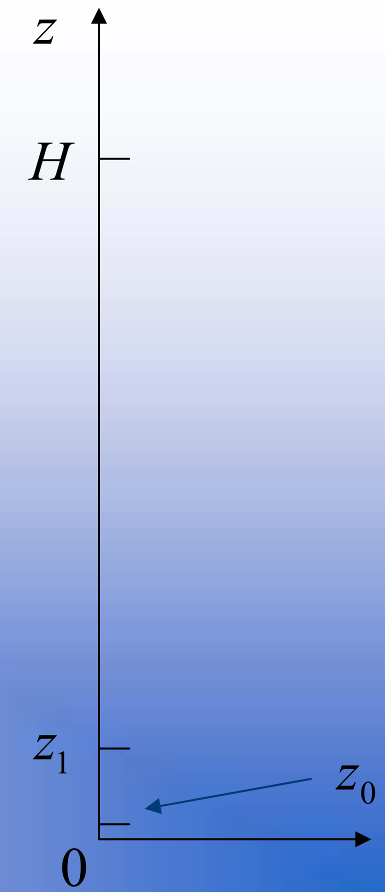
$$e = v_*^2 f_k(\xi_1) \qquad l = \kappa z f_l(\xi_1) \qquad \xi_1 = z / L$$

$$\Theta = \Theta_{obs}(t) \qquad Q = Q_{obs}(t)$$

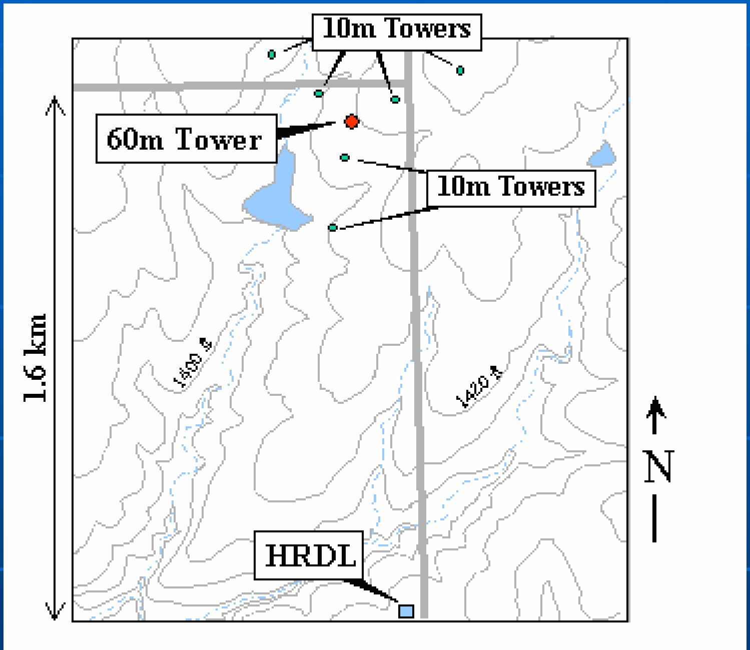
- $z = H$

$$\frac{\partial U}{\partial z} = \frac{\partial V}{\partial z} = \frac{\partial e}{\partial z} = \frac{\partial l}{\partial z} = \frac{\partial Q}{\partial z} = 0 \qquad \frac{\partial \Theta}{\partial z} = \gamma$$

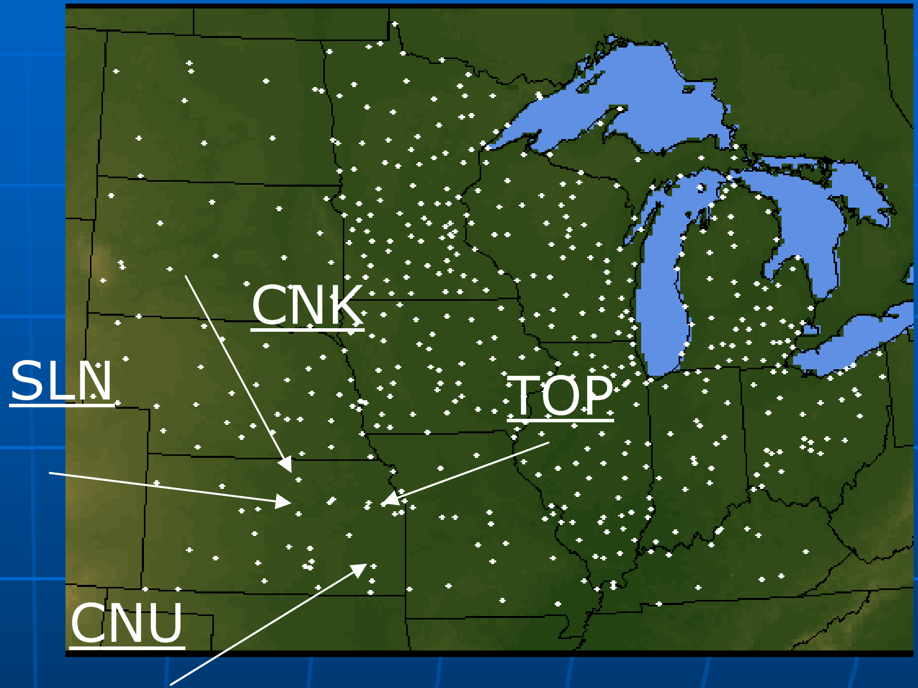
Here $z_0$ is the roughness height, $z_1$ is the height of the first computational level, and $H$ the height of the second computational level, $L$ is Monin–Obukhov scale.

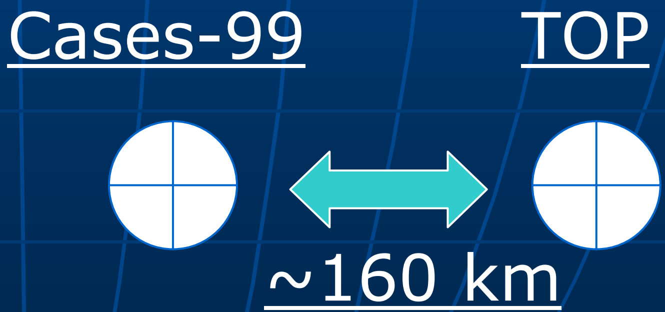


# CASES-99




The main CASES-99 field site showing the location of HRDL relative to the main 60 m tower. HRDL was positioned at an altitude of 431 m MSL, a latitude of 37.6360° N and a longitude of −96.7339°. The contour interval is 10 feet and the horizontal distance from HRDL to the main tower was 1.45 km.



# Top

- Station information and sounding indices
- Station identifier: TOP
- Observation time: 991005/0000
- Station latitude: 39.07
- Station longitude: -95.62
- Station elevation: 270.0

| PRES hPa | **HGHT** m | TEMP C | DWPT C | RELH % | **MIXR** g/kg | **DRCT** deg | **SKNT** knot | **THTA** K | THTE K | THTV K |
|---|---|---|---|---|---|---|---|---|---|---|
| 992.0 | 270 | 14.4 | 4.4 | 51 | 5.31 | 140 | 8 | 288.2 | 303.6 | 289.1 |
| 987.8 | 305 | 14.1 | 4.2 | 51 | 5.24 | 145 | 8 | 288.2 | 303.4 | 289.2 |
| 952.4 | 610 | 11.4 | 1.9 | 52 | 4.65 | 170 | 12 | 288.5 | 302.0 | 289.3 |
| 925.0 | 853 | 9.2 | 0.2 | 53 | 4.22 | 175 | 12 | 288.7 | 301.0 | 289.4 |
| 918.2 | 914 | 8.6 | -0.4 | 53 | 4.07 | 175 | 12 | 288.7 | 300.7 | 289.4 |
| 911.0 | 979 | 8.0 | -1.0 | 53 | 3.92 | 184 | 11 | 288.7 | 300.2 | 289.4 |
| 884.9 | 1219 | 7.9 | -7.0 | 34 | 2.58 | 215 | 8 | 291.0 | 298.8 | 291.5 |
| 871.0 | 1350 | 7.8 | -10.2 | 27 | 2.03 | 252 | 11 | 292.3 | 298.5 | 292.6 |
| 865.0 | 1407 | 9.0 | -17.0 | 14 | 1.17 | 267 | 13 | 294.1 | 297.9 | 294.3 |
| 852.9 | 1524 | 9.5 | -10.9 | 23 | 1.97 | 300 | 16 | 295.8 | 302.0 | 296.1 |
| 850.0 | 1552 | 9.6 | -9.4 | 25 | 2.22 | 280 | 12 | 296.2 | 303.1 | 296.6 |
| 822.1 | 1829 | 10.9 | -14.4 | 15 | 1.53 | 315 | 21 | 300.4 | 305.3 | 300.6 |
| 819.0 | 1861 | 11.0 | -15.0 | 15 | 1.46 | 314 | 21 | 300.8 | 305.6 | 301.1 |
| 792.2 | 2134 | 8.9 | -15.1 | 17 | 1.50 | 310 | 21 | 301.5 | 306.4 | 301.8 |
| 763.4 | 2438 | 6.7 | -15.2 | 19 | 1.54 | 305 | 23 | 302.2 | 307.3 | 302.5 |
| 735.6 | 2743 | 4.3 | -15.3 | 22 | 1.59 | 305 | 27 | 302.9 | 308.2 | 303.2 |
| 715.0 | 2976 | 2.6 | -15.4 | 25 | 1.62 | 311 | 26 | 303.5 | 308.8 | 303.8 |
| 700.0 | 3147 | 1.8 | -17.2 | 23 | 1.43 | 315 | 25 | 304.4 | 309.2 | 304.7 |
| 657.1 | 3658 | 1.8 | -34.6 | 5 | 0.31 | 305 | 27 | 310.0 | 311.1 | 310.1 |
| 651.0 | 3733 | 1.8 | -37.2 | 4 | 0.24 | 304 | 27 | 310.8 | 311.7 | 310.9 |
| 608.9 | 4267 | -1.6 | -32.2 | 8 | 0.42 | 295 | 29 | 312.9 | 314.5 | 313.0 |
| 582.0 | 4627 | -3.9 | -28.9 | 12 | 0.61 | 292 | 34 | 314.3 | 316.5 | 314.4 |

# CNK, CNU, SLN, TOP

| STN | TIME | PMSL | ALTM | TMP | DEW | RH | DIR | SPD | VIS | CLOUDS | MIN | MAX | SNO |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DD/HHMM | hPa | hPa | C | C | % | deg | m/s | km | | C | C | in |
| CNK | 06/2255 | 1009.9 | 1010.8 | 25 | 13 | 48 | 160 | 9 | | | | | |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | | | | | |
| CNK | 04/2355 | 1021.4 | 1021.0 | 15 | 1 | 39 | 180 | 5 | | | | | |
| CNU | 06/2254 | 1014.9 | 1015.6 | 24 | 11 | 43 | 170 | 6 | | | | | |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | | | | | |
| CNU | 04/2354 | 1024.4 | 1024.4 | 12 | 5 | 61 | 120 | 2 | | | | | |
| SLN | 06/2253 | 1010.7 | 1011.5 | 26 | 13 | 45 | 170 | 10 | | | | | |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | | | | | |
| SLN | 04/2353 | 1023.1 | 1022.7 | 15 | 2 | 42 | 170 | 4 | | | | | |
| TOP | 06/2256 | 1013.9 | 1014.2 | 24 | 11 | 45 | 140 | 6 | | | | | |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | | | | | |
| TOP | 04/2356 | 1024.1 | 1023.7 | 13 | 4 | 57 | 140 | 3 | | | | | |

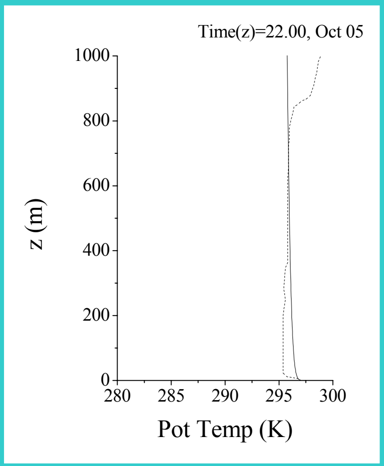
# Comparison of theory and experiment (Pot. Temp.)

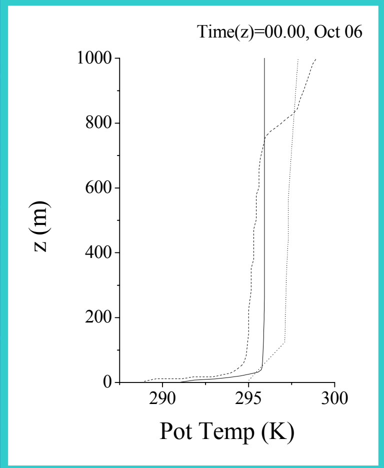
**Time(z)=22.00, Oct. 05, 1999**

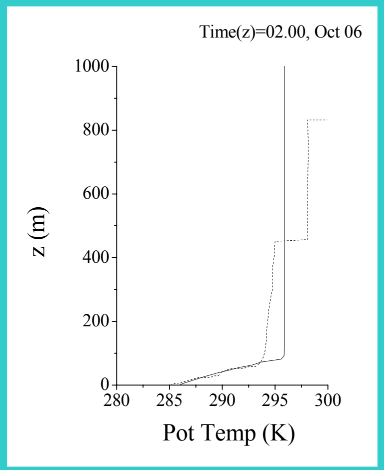
**Time(z)=00.00, Oct. 06, 1999**

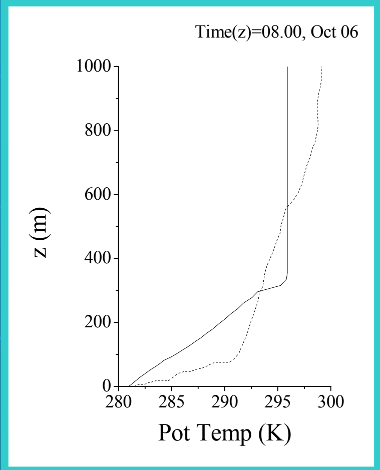
**Time(z)=02.00, Oct. 06, 1999**

**Time(z)=08.00, Oct. 06, 1999**




**_____ - Theory, - - - - - CASES-99, . . . . . - TOP**

# Comparison of theory and experiment (Speed)

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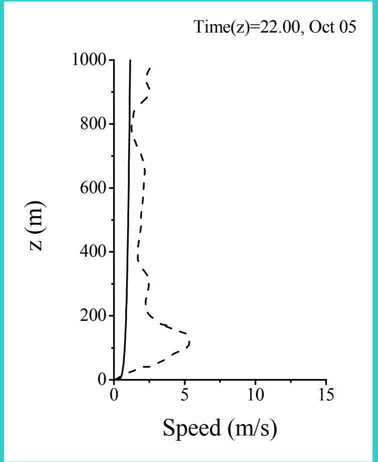
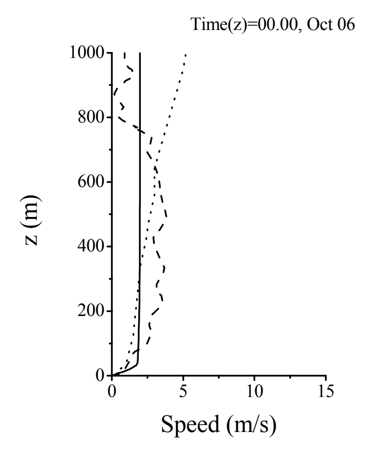
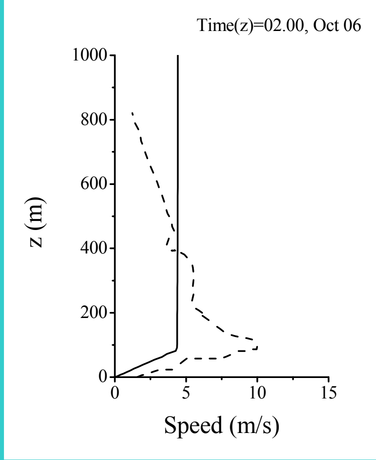
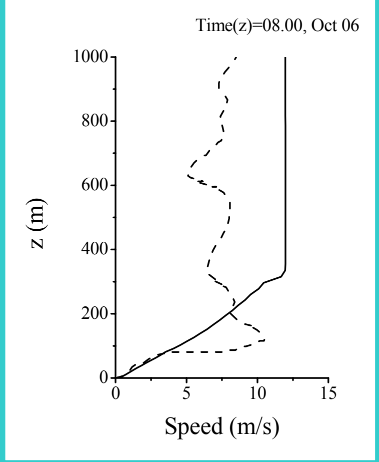
**Time(z)=02.00, Oct. 06, 1999**

**Time(z)=08.00, Oct. 06, 1999**



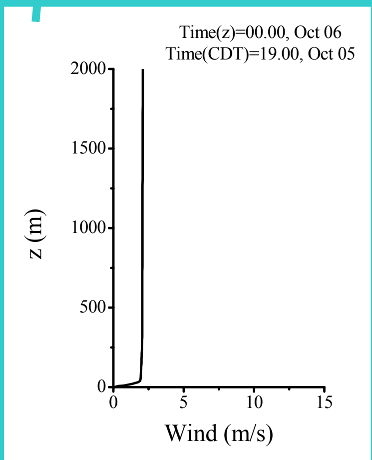



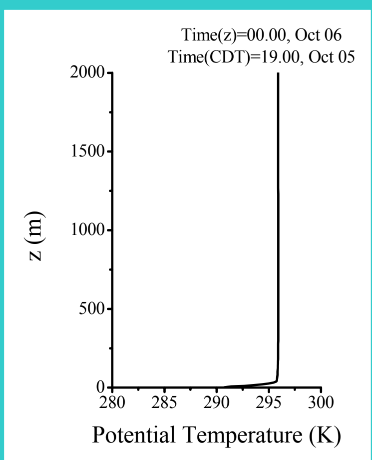




**_____ - Theory, - - - - - CASES-99, . . . . . - TOP**

# Numerical simulation
## (wind and potential temperature)

**Time(z)=00.00, Oct. 06, 1999**

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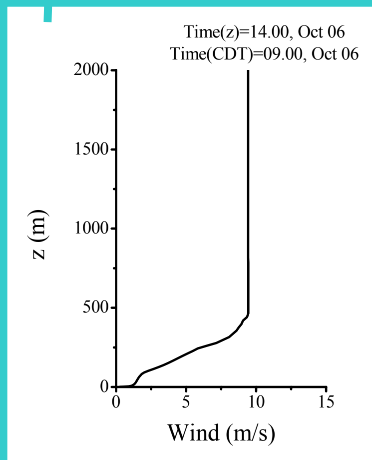
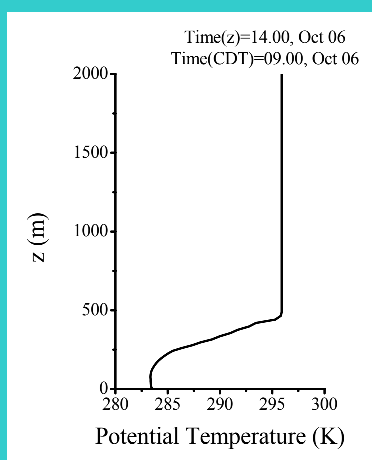
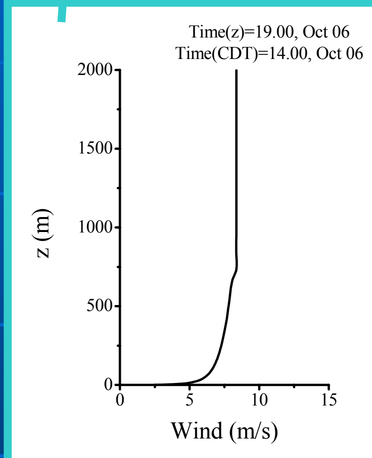
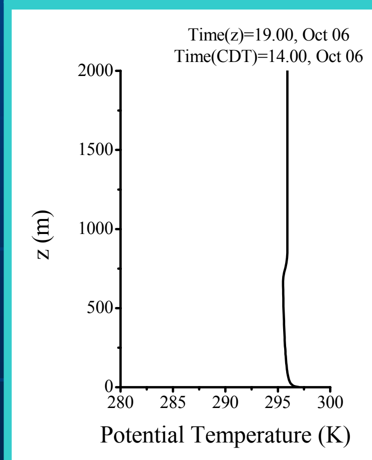



**Time(z)=19.00, Oct. 06, 1999**

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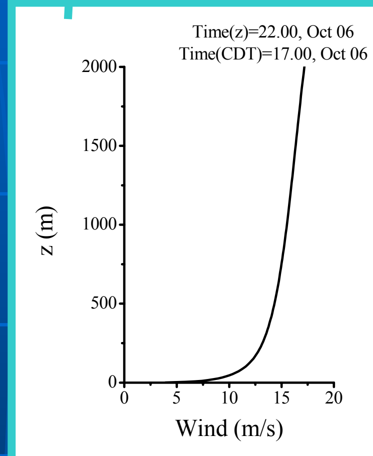


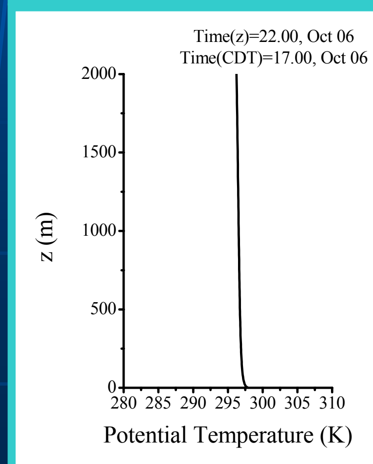

# Numerical simulation
## (TKE and TKE Rate)

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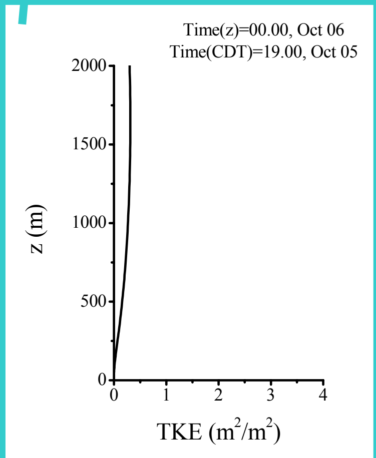


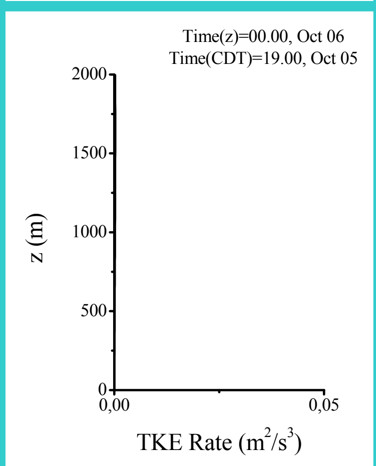


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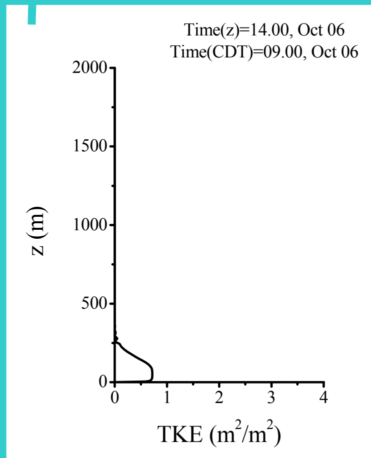


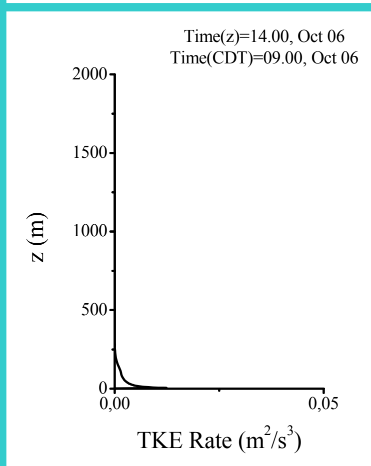


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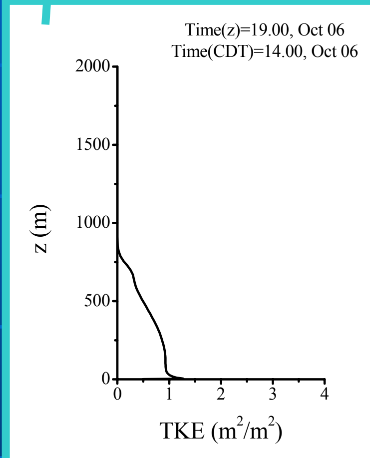
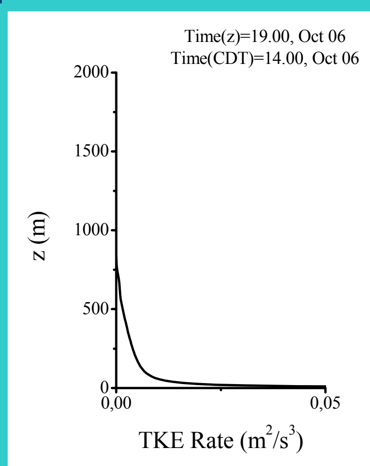
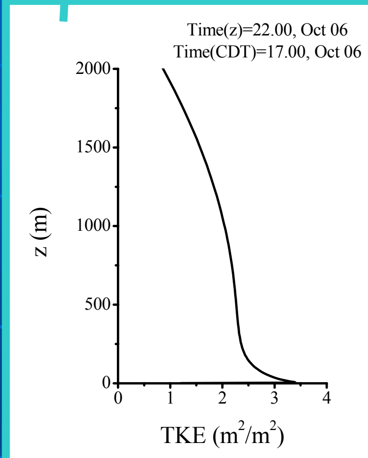
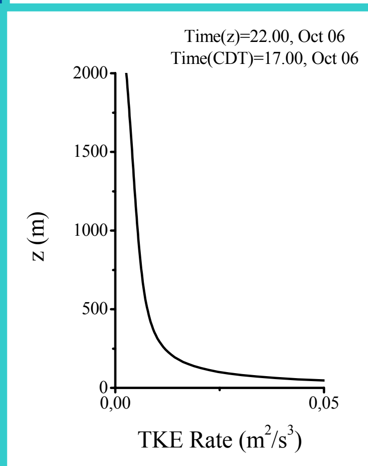


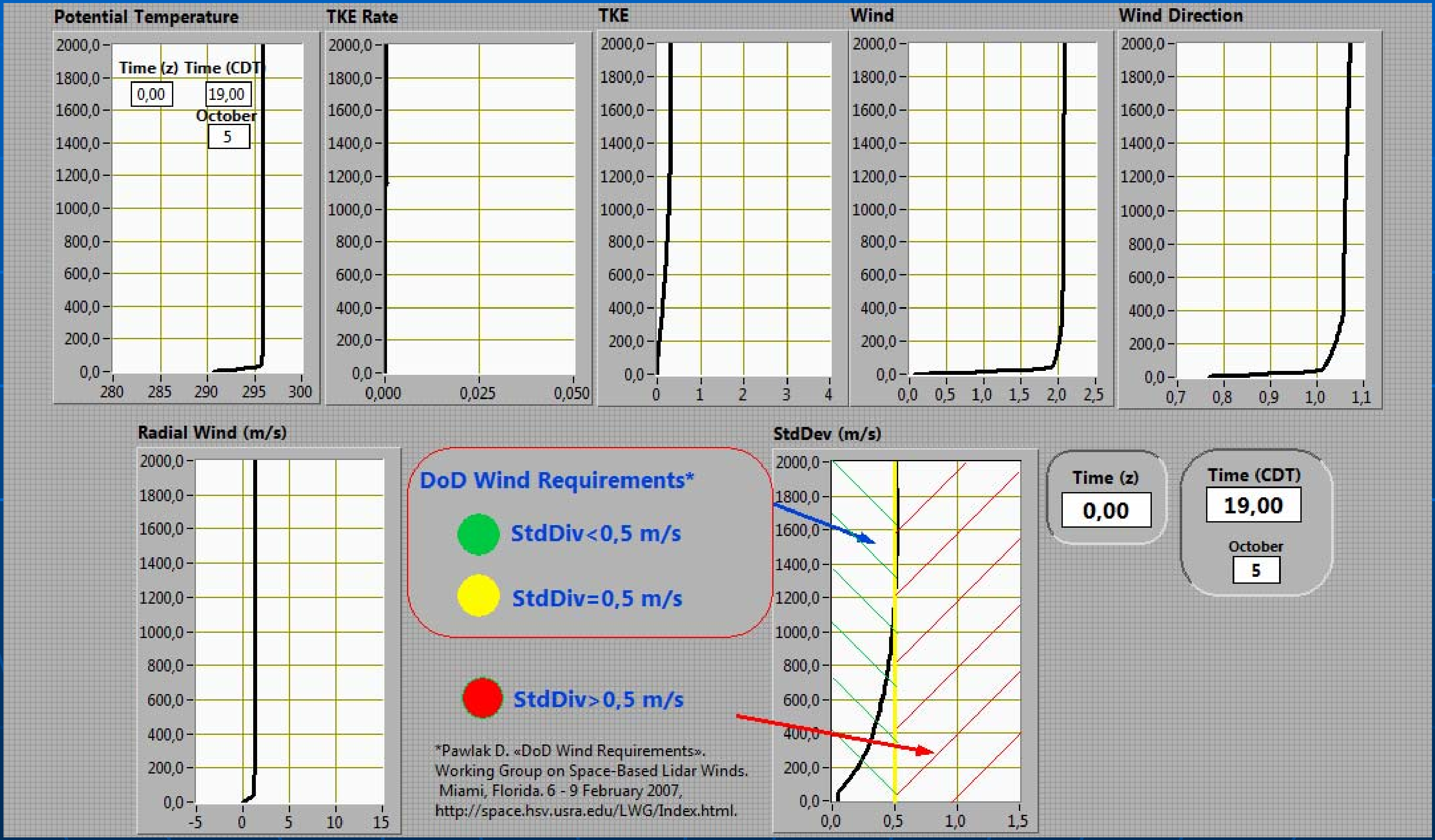

**Time(z)=22.00, Oct. 06, 1999**



# Numerical simulation

## (radial wind and standard deviation)



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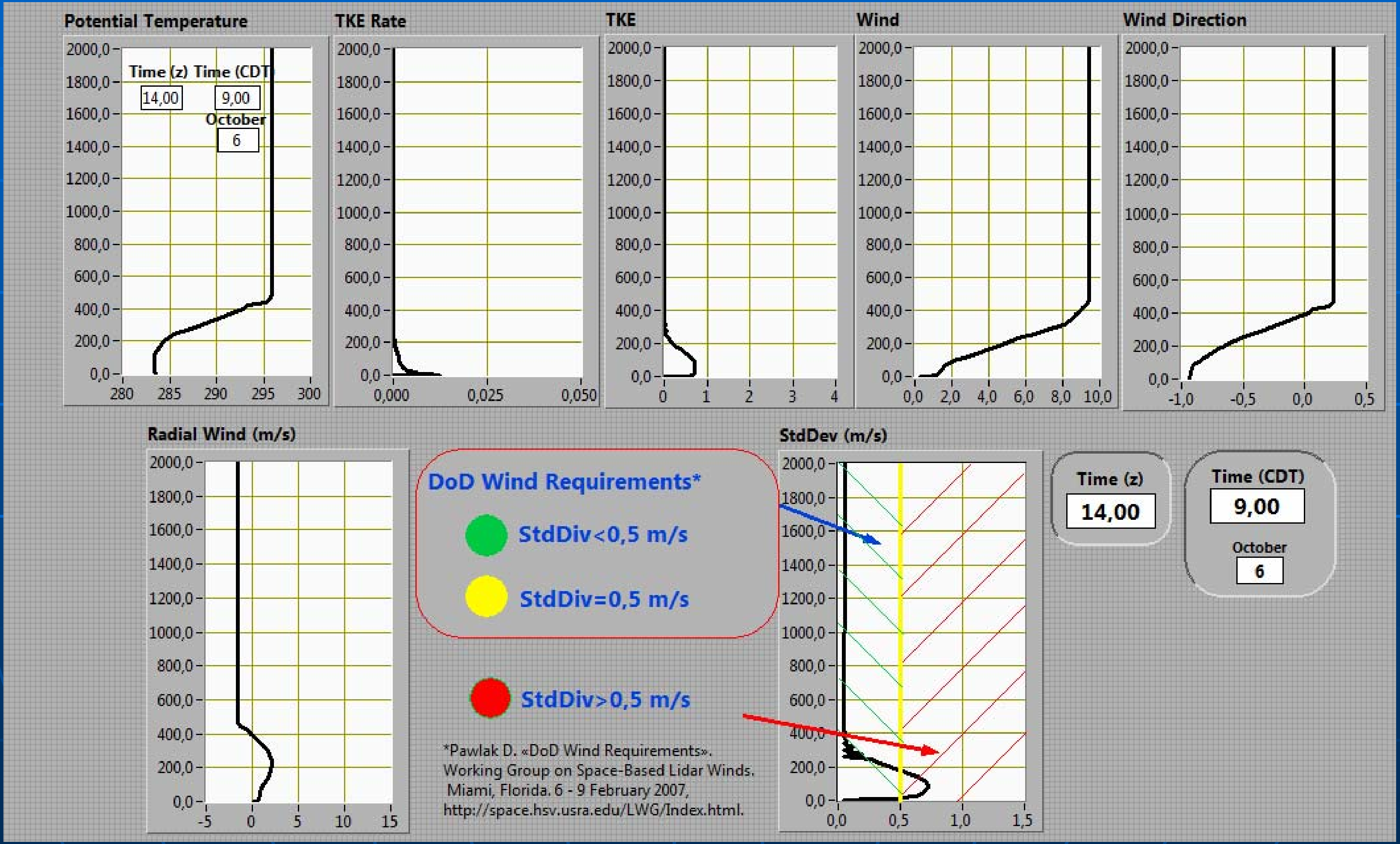

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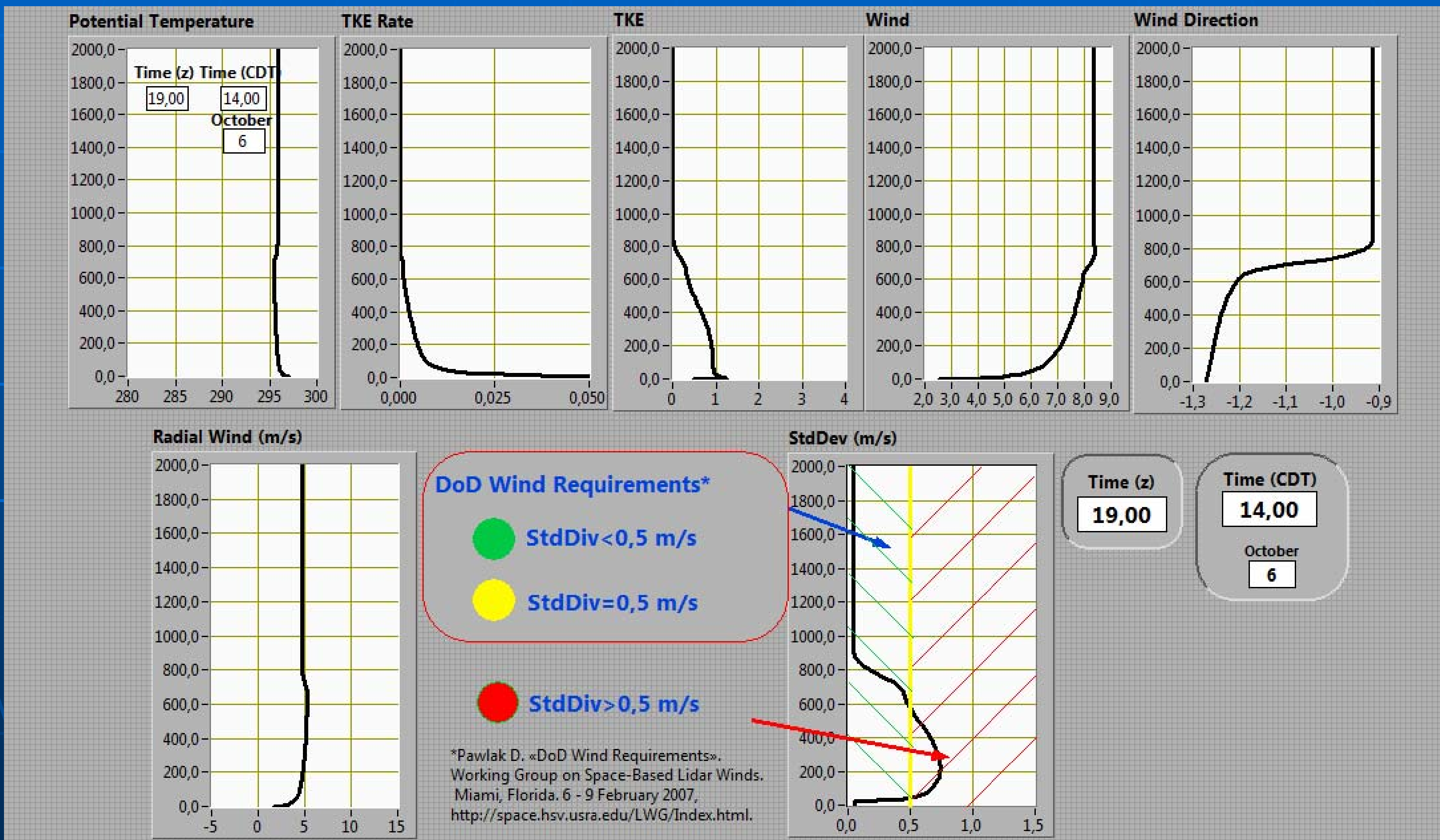

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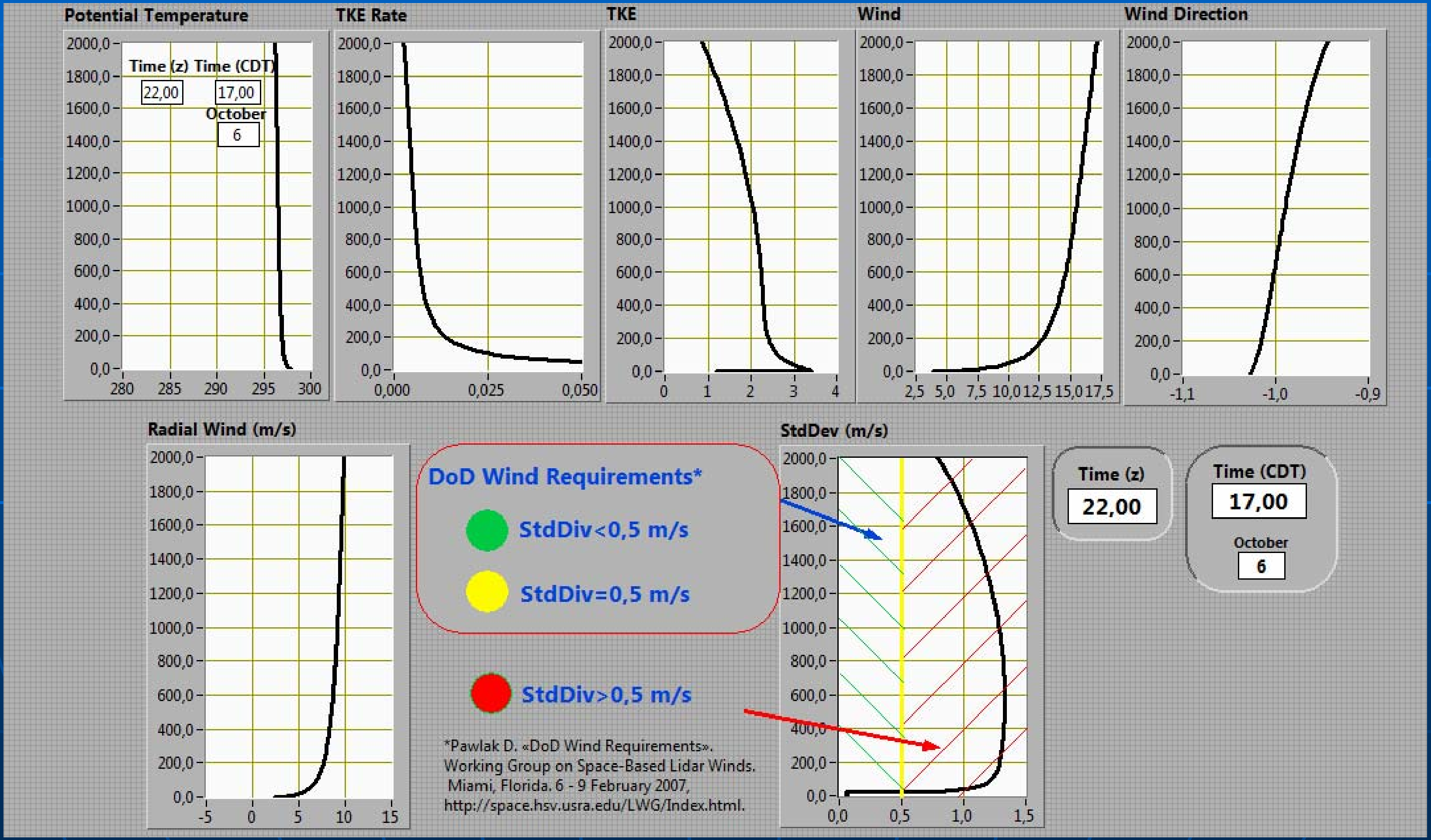

# Conclusions

- The precision of measurement of the radial wind velocity in PBL is simulated numerically based on analysis of the Doppler shift of a non-Gaussian random signal. For short-term prediction of the precision of measurement we use the 1D-model of the planetary boundary layer and the "e-l"-model of atmospheric turbulence, which take into account diurnal variations of meteorological parameters and the turbulent structure of the PBL.
- The numerical model of Doppler measurements is proposed for experiment CASES-99 using model of atmospheric turbulence corresponding to the level 2.5 of Yamada – Mellor classification. For interpretation of experimental data we use meteorological information from website of University of Wyoming from TOP, CNK, SLN, and CNU station. It is shown that the results of numerical simulation agree with the experimental data
- The standard deviation of random error of the radial wind velocity in PBL is studied for the different time of the day. It is shown that with the growth of turbulence (in the morning, during the day, and in the evening) the standard deviation increases. The value of standard deviation can surpass the value 0,5 m/s (DoD Wind Requirements: http://space.hsv.usra.edu/LWG/Index.htm).