# **GEOL 5690: Dynamic Topography**

## Defining dynamic topography

In a sense, all topography is dynamic as it is always changing thanks to erosional and depositional processes as well as tectonic activity at greater depths. Thus the term "dynamic topography" can seem trite. Over the last 20-30 years or so, "dynamic topography" in a geodynamic sense has taken off as an explanation for long-wavelength variations in elevation and consequences of such changes. Unfortunately the term in this context has varied interpretations. For instance, the decline in seafloor elevation with time since creation at a midocean ridge is sometimes considered to be "dynamic topography," while for others it is just isostatic and entirely a lithospheric process. Convective removal of dense lithosphere should produce an uplift; some would consider this dynamic topography and others would view it as being in isostatic equilibrium.

For the discussion here, dynamic topographic effects are changes in elevation produced by the transmission of viscous stresses to the lithosphere. In this context, seafloor sinking with age and removal of mantle lithosphere are considered to be isostatic. Deflections of a continent because of flow induced by subduction would be dynamic. Note that some ambiguity exists in a subduction zone where the load of a slab is isostatic when connected to the overriding lithosphere and dynamic once viscous mantle material intercedes. A more nuanced and complete discussion can be found in Molnar et al. (2015).

## Slab dip and topography

Sections 6-7 to 6-9 of Turcotte and Schubert lay out the background, section 6-11 has the payoff.

A puzzle that challenged early models of subduction was that the dip of slabs as observed in the Earth is not 90°. Slabs dip as low as 5° up to about 70° or so. While there is some influence from the thermal structure of the slab, one key element is the recognition that pressures from fluid flow are critical. This is also important as these fluid motions are being driven by the motion of the slab and so absorb a lot of the slab pull force. We first note that motion of a viscous fluid can be solved with the aid of a stream function  $\psi$ 

$$0 = \frac{\partial^4 \psi}{\partial x^4} + 2 \frac{\partial^4 \psi}{\partial x^2 \partial z^2} + \frac{\partial^4 \psi}{\partial z^4} = \nabla^4 \psi$$
(1)

where the velocities in the fluid are related to the stream function thus:

$$u = -\frac{\partial \psi}{\partial z}$$

$$v = \frac{\partial \psi}{\partial x}$$
(2)

For this problem, we posit a stream function of this form:

$$\psi = (Ax + Bz) + (Cx + Dz)\arctan\left(\frac{z}{x}\right)$$
 (3)

This can be verified as a solution following eqns. 6-108 and 6-109 in Turcotte and Schubert. Substitution into the equations for velocity yield

$$u = -B - D\arctan\frac{z}{x} + (Cx + Dz)\left(\frac{-x}{x^2 + z^2}\right)$$

$$v = A + C\arctan\frac{z}{x} + (Cx + Dz)\left(\frac{-z}{x^2 + z^2}\right)$$
(4)

The pressure can be found by using one of these equations in our force balance equations

$$0 = -\frac{\partial p}{\partial x} + \eta \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right)$$
  
$$0 = -\frac{\partial p}{\partial z} + \rho g + \eta \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right)$$
(5)

and find

$$P = \frac{-2\eta(Cx + Dz)}{x^2 + z^2} \tag{6}$$

We then have to apply continuity boundary conditions: no velocity along the base of the overriding plate and velocity equal to that of the descending slab along its top (along a line of constant arctan). This is done for a 45° dipping slab in Turcotte and Schubert with the result that

$$P = \frac{4\eta U}{(2 - \pi^2/4)r} = \frac{-8.558\eta U}{r}$$
(7)

where r is measured down along the slab top and U is the velocity of the slab. Because the force decreases exactly with distance, the moment per unit length of slab remains the same. A similar analysis on the bottom of the slab reveals a comparable force also pointing upwards with magnitude only about 5% of the inside corner flow (0.462 vs. 8.558). In this formulation, the torque applied by these dynamic pressures balances the torque from the slab trying to pivot downward from the trench.

The dynamic pressure along the base of the overriding plate also falls out:

$$P = \frac{\eta \pi U \sqrt{2}}{x \left(2 - \pi^2 / 4\right)} = \frac{-3.026 \eta U}{x}$$
(8)

If we balance the pressure by depressing the surface, this would be the dynamic topography associated with the subducting slab. This solution has the unhappy property of being infinite near the corner of flow. Note too that this is assuming a constant viscosity. Presence of a low-viscosity channel (crust or asthenosphere) will tend to drive flow in the channel towards the slab

and reduce the pressure on the base of the plate. In general, both pressures will increase as the slab dip shallows.

(This same physics can apply to the evolution of a subduction channel if wedge shaped; see Moulas et al., 2021 for how this handcuffs some models of flow into a subduction zone).

The alternative means of modelling this is simply to assume that the slab is falling into the mantle and the overriding plate is moving to keep up (see below); often these solutions are kinematically imposed and the far-field boundary conditions are a challenge. This is in a sense the approach in Mitrovica et al. (1989) and seems to be the basis for the "tomotectonics" of Clennett et al. (2020), which was challenged as not physically consistent with broader mantle flows by Li et al. (2023). Thus dynamic stresses from such models should be treated with care.

Although this analytic solution has long been popular and gives us some useful insight into the problem, a recent unpublished analysis comparing these predictions against modern numerical models of subduction suggests that ignoring the internal deformation of the slab results in noticeable discrepancies between the numerical models and the analytical solution. The singularity in the corner where the slab and overriding plate and asthenosphere meet is another limitation. Other issues arise with the development of torques at major phase transitions (basalt/eclogite, olivine  $\alpha$ - $\beta$ - $\gamma$  transitions). So these equations only give us some insight into the processes acting on slabs.

### Sinking or rising blobs

A conceptually simpler case is if we have an isolated blob of material rising or sinking in a viscous medium. The movement of the blob will induce movement of the fluid, which will include a change in the topography at the surface of the medium. This was investigated by Morgan (1965) and modified some by Chatelain et al. (1992); we'll follow the development in the appendix of Molnar et al. (2015) that follows.

The analysis presented here both extends the solution that *Morgan* [1965] gave for surface deflection and gravity anomalies to include terms that he discarded, those accurate to terms in  $a^2/D^2$  in Taylor expansions (see figure for a definition of terms), and also considers a viscous or inviscid sphere, in addition to his treatment for a rigid sphere.



First, a sphere rising through a viscous fluid of infinite extent can be written as

$$V = -\frac{g\delta\rho a^2}{3f\eta} \tag{9}$$

where

$$f = \frac{\eta + \frac{3}{2}\eta_{sphere}}{\eta + \eta_{sphere}}$$
(10)

and  $\eta_{sphere}$  is the viscosity of the sphere [*Batchelor*, 1967, p. 236]. Clearly, when the sphere is rigid,  $\eta_{sphere} >> \eta$ , f = 3/2, and (9) reduces to the usual Stokes-flow result with a 1/3f = 2/9. In spherical coordinates with the center of the sphere defining the origin, the components of velocity of the fluid surrounding the sphere, which is at rest at infinite distance are:

$$v_R(R,\theta) = V\cos\theta \left[ f\left(\frac{a}{R} - \frac{a^3}{R^3}\right) + \frac{a^3}{R^3} \right]$$
(11)

$$v_{\theta}(R,\theta) = -V\sin\theta \left[\frac{f}{2}\left(\frac{a}{R} + \frac{a^3}{R^3}\right) - \frac{a^3}{2R^3}\right]$$
(12)

[e.g., Griffiths, 1986].

*Morgan* [1965] showed that the influence of a surface at which the vertical component of velocity must vanish introduces a factor that reduces the speed of the sphere within the fluid, which we write here as

$$V_{\varepsilon} = \frac{V}{1 + \frac{3\varepsilon}{4} + \frac{9\varepsilon^2}{16} \dots} = -\frac{g\delta\rho a^2}{3f\eta} \frac{1}{1 + \frac{3\varepsilon}{4} + \frac{9\varepsilon^2}{16} \dots}$$
(13)

where  $\varepsilon = a/D$ .

We need boundary conditions on the surface, which we express in cylindrical coordinates, where *r* is the radial distance from an origin centered over the center of the sphere, *D* is its depth, and  $R^2 = D^2 + r^2$ . With these coordinates,  $\cos \theta = D/R$ , and  $\sin \theta = r/R$ . From (11) and (12), we obtain

$$v_{r}(r) = V \sin\theta \cos\theta \left\{ \left[ f\left(\frac{a}{R} - \frac{a^{3}}{R^{3}}\right) + \frac{a^{3}}{R^{3}} \right] - \left[ \frac{f}{2} \left(\frac{a}{R} + \frac{a^{3}}{R^{3}}\right) - \frac{a^{3}}{2R^{3}} \right] \right\}$$
  
=  $VaDr \left[ \frac{f}{R^{3}} - 3(f-1)\frac{a^{2}}{R^{5}} \right]$  (14)

This twice what we would infer from (11) and (12), but Morgan's solution exploits images. Hence, there are effectively two spheres, one on either side of the boundary.

To calculate the deflection of the surface, we must estimate the vertical normal stress on the surface, which consists of a perturbation to the pressure due to the flow and the vertical normal deviatoric stress.

$$\sigma_{zz} = -p + 2\eta\tau_{zz} = -p + 2\eta\frac{\partial v_z}{\partial z}$$
(15)

He showed that the perturbation pressure surrounding the sphere is

$$p(r) = 3Va\eta \frac{D}{R^3} \tag{16}$$

We use incompressibility to estimate the vertical normal deviatoric stress:

$$\tau_{zz} = 2\eta \frac{\partial v_z}{\partial z} = -2\eta \frac{1}{r} \frac{\partial r v_r}{\partial r}$$
(17)

This leads to:

$$-2\eta \frac{1}{r} \frac{\partial (rv_r)}{\partial r} = -\frac{2\eta V a D}{r} \frac{\partial}{\partial r} \left( r^2 \left[ \frac{f}{R^3} - 3(f-1) \frac{a^2}{R^5} \right] \right)$$

$$= 2\eta V a D \left\{ \frac{f}{R^3} - \frac{9(f-1)a^2}{R^5} - \frac{3fD^2}{R^5} + \frac{15(f-1)a^2D^2}{R^7} \right\}$$
(18)

So, the total normal stress becomes:

$$\sigma_{zz} = -3Va\eta \frac{D}{R^3} + 2\eta VaD \left[ \frac{f}{R^3} - \frac{9(f-1)a^2}{R^5} - \frac{3fD^2}{R^5} + \frac{15(f-1)a^2D^2}{R^7} \right]$$

$$= -VaD\eta \left[ \frac{3-2f}{R^3} + \frac{18(f-1)a^2}{R^5} + \frac{6fD^2}{R^5} - \frac{30(f-1)a^2D^2}{R^7} \right]$$
(19)

The surface uplift, in turn becomes:

$$\delta h(r) = -\frac{\sigma_{zz}}{\rho_m g}$$

$$= \frac{VaD\eta}{\Delta\rho g} \left[ \frac{3-2f}{R^3} + \frac{18(f-1)a^2}{R^5} + \frac{6fD^2}{R^5} - \frac{30(f-1)a^2D^2}{R^7} \right] \qquad (20)$$

$$= -\frac{\delta\rho a^3}{3f\Delta\rho} \frac{D}{R^3} \left[ 3-2f + \frac{18(f-1)a^2}{R^2} + \frac{6fD^2}{R^2} - \frac{30(f-1)a^2D^2}{R^4} \right]$$

where  $\Delta \rho$  is the density contrast of the topography (here taken to be the top of the mantle relative to air, so 3300 kg m<sup>-3</sup>). This is plotted below for a rigid sphere, f = 3/2, in panel (a) for three different geometries. In his simpler solution, equation (15) of *Morgan* [1965], the first term vanishes because f = 3/2, and he included only the third of the four terms in (20).

We might consider the value of uplift (or subsidence) directly above the sphere (r=0) by setting R=D:

$$\delta h(r=0) = -\frac{\delta \rho a^3}{3f \Delta \rho} \frac{1}{D^2} \left[ 3 + 4f - \frac{12(f-1)a^2}{D^2} \right]$$
(20a)

When *a* is noticeably smaller than *D*, we can see that the amplitude really depends on the lefthand term; reducing to Morgan's (1965) solution by assuming a solid sphere (f = 3/2) whose radius is sufficiently smaller than the sphere's depth yields

$$\delta h(r=0) = -\frac{\delta \rho a^3}{\Delta \rho} \frac{2}{D^2}$$

$$= -\frac{3M}{2\pi \Delta \rho D^2}$$
(20b)

where M is the total mass surplus or deficit of the sphere relative to the surrounding material.



Figure A5 from Molnar et al. (2015). (a) Surface height (km), (b) free-air gravity anomaly (mGal), and (c) ratio of free-air gravity anomaly to surface height (mGal/km) versus distance from the point directly over the center of the rising sphere (Figure A4) for three cases: a sphere of radius 500 km and centered at depths of 1000 (red) and 2000 km (blue), and a sphere of radius 1000 km centered at a depth of 2000 km (black). The density of the sphere is chosen so that the deflection directly over the sphere is 1 km. Not surprisingly, when the sphere is small in radius (500 km) and shallow (1000 km) in depth (red), the deflection is more localized than when it is deep.

Basically, in this formulation, the deflection is mainly due to the mass involved and it decreases with the square of the depth. A line load (more like a slab) would tend to decrease more slowly with depth. If we imagine a dipping slab as a series of blobs descending at different depths, we can see how shallowing a slab's dip will cause subsidence to move inland as the blob under a given part of the foreland becomes more shallow. In a sense, this approach was used by Mitrovica et al. (1989) to argue for a dynamic origin to subsidence producing the Interior Seaway that linked the Gulf of Mexico with the Arctic Ocean:



The figure superimposes the temperature field (and thus density variations) from three sinking blobs offset horizontally to make a  $\sim 45^{\circ}$  "slab".

However, one certain complication is that the Earth's viscosity is not constant. So while the viscosity of the halfspace does not enter into the equations above, if there are variations, those variations will have an impact. Additionally, there are density discontinuities in the earth that can complicate the rise or descent of a density anomaly. So while these calculations give us a sense of the magnitude of possible deflections, more thorough analyses are needed to give quantitative estimates of dynamic topography. However, Morgan (1965) showed that even placing a relatively low viscosity layer above a stiffer layer tended to only reduce the surface deflections a small amount. More complex geometries (e.g., low viscosity in the counterflow above a subducting slab) can have more profound effects on dynamic topography.

**Gravity and dynamic topography**. One of the points of disagreement about the amplitude of modern dynamic topography is whether other geophysical observations are consistent with claims from some models of as much as 2-3 km of dynamic topography. One such observable is free-air gravity anomalies (which we'll discuss a bit more later in the class).

The gravity anomaly produced by the simple sphere-in-a-uniform-halfspace consists of a part due to the surface deflection and that due to the mass of the sphere:

$$\Delta g(r) = 2\pi G \Delta \rho \delta h + G \frac{4\pi \delta \rho a^3}{3} \frac{D}{R^3}$$
(21)

Inserting (20) into (21) yields:

$$\Delta g(r) = -2\pi G \frac{\delta \rho a^3 D}{3f} \left[ \frac{3-2f}{R^3} + \frac{18(f-1)a^2}{R^5} + \frac{6fD^2}{R^5} - \frac{30(f-1)a^2D^2}{R^7} \right] + \frac{4\pi G \delta \rho a^3}{3} \frac{D}{R^3} = -\frac{4\pi G \delta \rho a^3}{3f} \frac{D}{R^3} \left[ \frac{(3-4f)}{2} + \frac{9(f-1)a^2}{R^2} + \frac{3fD^2}{R^2} - \frac{15(f-1)D^2a^2}{R^4} \right]$$
(22)

which is plotted in the figure panel (b), for f = 3/2.

Finally, note that in the limit of  $a \ll D$  and at r = 0,

$$\frac{\Delta g}{\delta h} = \frac{-\frac{4\pi G \delta \rho a^3}{3f} \frac{D}{R^3} \left[ \frac{(3-4f)}{2} + \frac{9(f-1)a^2}{R^2} + \frac{3fD^2}{R^2} - \frac{15(f-1)D^2a^2}{R^4} \right]}{-\frac{\delta \rho a^3}{3f\Delta \rho} \frac{D}{R^3} \left[ 3-2f + \frac{18(f-1)a^2}{R^2} + \frac{6fD^2}{R^2} - \frac{30(f-1)a^2D^2}{R^4} \right]}{R^4}$$
(23)
$$= 4\pi G \Delta \rho \frac{\left[ \frac{(3-4f)}{2} + 3f \right]}{[3-2f+6f]} = 2\pi G \Delta \rho \frac{3+2f}{3+4f}$$

For a rigid sphere, f = 3/2, this simplifies to  $\frac{\Delta g}{\delta h} \approx \frac{4\pi G \Delta \rho}{3}$ , or 92 mGal/km of elevation if the density contrast being lifted is the mantle at 3300 kg m<sup>-3</sup>. For an inviscid sphere (f = 1), (23) becomes  $\Delta g/\delta h = 10\pi G \Delta \rho/7$ , which is only 9.5% greater than the solid sphere case.

In general, free-air gravity anomalies are not that large, and Molnar et al. (2015) argued that this placed an upper bound on modern dynamic topography of a few hundred meters. In this analysis, the anomaly from the causative mass is negative and it is the gravitational attraction of the uplifted surface, roughly a factor of three more than the sphere at depth, that dominates. In the plot below, for the case of a sphere of radius 500 km at a depth of 2000 km, the black line is the total free-air gravity anomaly and the red line is the contribution from a rising (low density) sphere driving the uplift.



What this suggests is that dynamic topography can hide if the deflection is somehow reduced relative to the value predicted from a uniform halfspace. The most obvious solution would be a low-viscosity layer; however, Morgan's (1965) analysis showed that it would have to be a very low viscosity layer. This is considered more fully in Molnar et al. (2015) as well as several other papers on dynamic topography.

#### References

- Batchelor, G. K., 1967, *An Introduction to Fluid Mechanics*, 615 pp., Cambridge Univ. Press, Cambridge, U. K.
- Chatelain, J.-L., Molnar, P., Prévot, R., and Isacks, B.L., 1992, Detachment of part of the downgoing slab and uplift of the New Hebrides (Vanuatu) Islands: Geophysical Research Letters, v. 19, p. 1507–1510.
- Clennett, E. J., Sigloch, K., Mihalynuk, M. G., Seton, M., Henderson, M. A., Hosseini, K., Mohammadzaheri, A., Johnston, S. T., and Müller, R. D., 2020, A Quantitative Tomotectonic Plate Reconstruction of Western North America and the Eastern Pacific Basin: Geochemistry, Geophysics, Geosystems, v. 20, no. 8, p. 25, doi: 10.1029/2020GC009117.
- Griffiths, R. W., 1986, Particle motions induced by spherical convective elements in Stokes flow, J. Fluid Mech., 166, 139–159.
- Li, Y. C., Liu, L. J., Peng, D. D., Dong, H., and Li, S. Z., 2023, Evaluating tomotectonic plate reconstructions using geodynamic models with data assimilation, the case for North America: Earth-Science Reviews, v. 244, 104518. doi: 10.1016/j.earscirev.2023.104518.
- Mitrovica, J.X., Beaumont, C., and Jarvis, G.T., 1989, Tilting of continental interiors by the dynamical effects of subduction: Tectonics, v. 8, p. 1079–1094.
- Molnar, P., England, P.C., and Jones, C.H., 2015, Mantle dynamics, isostasy, and the support of high terrain: Journal of Geophysical Research: Solid Earth, v. 120, p. 1932–1957, doi: 10.1002/2014JB011724.

- Morgan, W.J., 1965, Gravity anomalies and convection currents: 1. A sphere and cylinder sinking beneath the surface of a viscous fluid: Journal of Geophysical Research: Solid Earth, v. 70, p. 6175–6187.
- Moulas, E., Brandon, M. T., Hammon, J. D. V., and Schmalholz, S. M., 2021, On backflow associated with oceanic and continental subduction: Geophysical Journal International, v. 227, no. 1, p. 576-590, doi: 10.1093/gji/ggab246.