GEOL 5690: Plate reconstructions

Reference: Sleep and Fujita, *Principles of Geophysics*, sections 7.4 and 7.5 cover velocity (infinitesimal) and finite rotations.

The key element in plate reconstructions is the use of poles of rotation (often called Euler poles, though this is strictly inaccurate). Note that we will be using finite rotations, which sometimes don't follow some of the rules for infinitesimal rotations or rotation rates often used in geodetic applications.

First start with the recognition that the displacement of a body constrained to remain on the surface of a sphere can be described by the rotation of that body about an axis through the Earth's center with a specified angle ω. The intersection of the axis with the Earth's surface is the pole of rotation (there are two). This can be parameterized in polar coordinates as a vector Ω of length ω (in radians) with angles equal to the latitude and longitude corresponding to the pole. (Note that the polarity of ω reverses for the opposite pole). Generally, a rotation is positive when it is counterclockwise while looking towards the center of the Earth from the pole, but conventions do vary.

When dealing with velocities, the rotation rate Ω' can be crossed with the vector $\vec{X}$ to a point at the surface from the earth's center to get the absolute velocity of the point at $\vec{X}$. Note that for finite rotations, $\Omega \times \vec{X}$ yields a vector of a length $\omega X \sin\alpha$, where α is the angle between the pole and $\vec{X}$ and X is the length of $\vec{X}$. This is equal to the displacement of the point along the surface of the sphere, but the vector itself does not point to the position of the point after the rotation. Instead, the math gets a bit uglier, and in vector notation, the vector $\vec{S}$ of the new location of the point that was at $\vec{X}$ is found to be:

$$\vec{S} = \frac{(\vec{X} \bullet \Omega)\Omega}{\omega^2} + \left(\vec{X} - \frac{(\vec{X} \bullet \Omega)\Omega}{\omega^2} \right)\cos\omega + \frac{\Omega}{\omega} \times \vec{X} \sin\omega \tag{1}$$

This is rather awkward, and so instead we often will use matrix math to obtain the position of a point. If our rotation tensor **R** describes the rotation, then $\vec{X}_{new} = \mathbf{R}\vec{X}_0$. The form of **R** is itself tedious but readily programmed:

$$\mathbf{R} = \begin{bmatrix} x^2(1-\cos\omega)+\cos\omega & xy(1-\cos\omega)-z\sin\omega & xz(1-\cos\omega)+y\sin\omega \\ xy(1-\cos\omega)+z\sin\omega & y^2(1-\cos\omega)+\cos\omega & yz(1-\cos\omega)-x\sin\omega \\ xz(1-\cos\omega)-y\sin\omega & yz(1-\cos\omega)+x\sin\omega & z^2(1-\cos\omega)+\cos\omega \end{bmatrix} \tag{2}$$

where the vector (*x,y,z*) is the Cartesian coordinates of the unit vector pointing along Ω:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\lambda\cos\theta \\ \sin\lambda\cos\theta \\ \sin\theta \end{pmatrix} \tag{2a}$$

where λ is longitude and θ is latitude of the pole. Note that the inverse of the rotation matrix is its transpose (which is also the rotation matrix when negating ω).

An absolute rotation is nearly irrelevant in tectonics (though there have been numerous attempts to try to define an absolute frame of reference). Instead we specify motion of points on one plate with respect to the points on a second plate. Even this is vague, for we need a time frame. To fully disambiguate things, let us use call the rotation that moves plate B from its position at time 1 to its position at time 2 relative to plate A $\Omega_{AB}^{1\to 2}$ and so the associated rotation tensor is $\mathbf{R}_{AB}^{1\to 2}$. So a point $\vec{X}(1)$ (relative to plate A) on plate B at time 1 will move to $\vec{X}(2)=\mathbf{R}_{AB}^{1\to 2}\vec{X}(1)$, again relative to plate A.

For a plate reconstruction, we observe magnetic anomalies on two plates that were once at the same place. In practice, we cannot actually match point to point on each anomaly with the exception of the points where the anomalies intersect transform faults. Thus we seek to locate the pairs of anomaly-transform intersections on either side of a ridge and determine the rotation that maps one set back onto the other. This is a non-trivial inversion as for any given pair of points, there is an infinite number of rotations that will map one point onto the other (the poles lying on the great circle bisecting the two points). Sometimes the transforms themselves are used, as they should describe small circles about the rotation pole, but often there are issues with the geometry of the transforms that limit their usefulness. Introducing uncertainty in the location of the transform-anomaly intersection adds yet more difficulty (and has often been dispensed with). While we should be aware of these difficulties (which are discussed in Stock and Molnar, *Tectonics*, 1988), let us presume them solved and for any magnetic anomaly A(t), and so we have $\mathbf{R}_{AB}^{A(t)\to 0}$, which is the rotation that moves plate B from its position at time A(t) to its modern position. This rotation matrix represents a finite rotation about a pole with a finite rotation angle.

These are the initial rotations that are usually reported in the literature for pairs of plates separated by a spreading center. We usually find them limiting for two reasons: (1) we often desire to know the instantaneous relative motion of these two plates past one another and (2) we are often interested in the motion of plates separated by a destructive or transform boundary). The instantaneous motion would be trivial to determine if the position of the Euler pole remained the same through time, but in fact this is rarely the case (and it can be shown that it should not be the case: on a planet with three plates, if you hold the position of the Euler poles of two pairs of plates constant, it can be demonstrated that the position of the other pole cannot remain constant provided the poles are not all colinear).

To obtain the stage pole (to pole of rotation describing motion over a time interval in the past), we first note that the modern position of a point $\vec{X}$ on plate B is simply the product of a rotation matrix from time 1 to the present acting on the original position $\vec{X}(1)$:

$$\vec{X}(0)=\mathbf{R}_{AB}^{1\to 0}\vec{X}(1) \tag{3}$$

But of course, the point was first rotated from its position at time 1 to a position at time 2:

$$\vec{X}(2)=\mathbf{R}_{AB}^{1\to 2}\vec{X}(1) \tag{4}$$

and in turn from time 2 to the present

$$\begin{aligned}\vec{X}(0) &= \mathbf{R}_{AB}^{2\to 0}\vec{X}(2)\\ &= \mathbf{R}_{AB}^{2\to 0}\left[\mathbf{R}_{AB}^{1\to 2}\vec{X}(1)\right]\end{aligned} \tag{5}$$

If we combine equations 3 and 5, we get

$$\begin{aligned}\mathbf{R}_{AB}^{1\to 0}\vec{X}(1) &= \mathbf{R}_{AB}^{2\to 0}\left[\mathbf{R}_{AB}^{1\to 2}\vec{X}(1)\right]\\ \mathbf{R}_{AB}^{1\to 0} &= \mathbf{R}_{AB}^{2\to 0}\mathbf{R}_{AB}^{1\to 2}\\ \mathbf{R}_{AB}^{0\to 2}\mathbf{R}_{AB}^{1\to 0} &= \mathbf{R}_{AB}^{0\to 2}\mathbf{R}_{AB}^{2\to 0}\mathbf{R}_{AB}^{1\to 2}\\ \mathbf{R}_{AB}^{0\to 2}\mathbf{R}_{AB}^{1\to 0} &= \mathbf{R}_{AB}^{1\to 2}\end{aligned} \tag{6}$$

Note that the rotation of a point from its position at time 2 to its position at time 0 followed by a rotation from its position at time 0 to its position at time 2 (which is the rotation about the same pole with an opposite polarity for the rotation angle) yields no motion at all (the identity matrix), so those terms cancel and we have an equation allowing us to obtain the net rotation of plate B relative to plate A from time 1 to time 2 from the modern measurements of rotations of the different magnetic anomalies. (As an aside, note that the rotation that moves B relative to A over time from 1 to 2 is not trivially related to the pole that moves A relative to B over the same time: this is because the pole positions are relative to the fixed plate and so generally these two rotations will not share the Euler pole position).

The second problem is solved by finding a series of connections between our two plates A and B and using a series or known rotations to recover the unknown rotation. So let us say that there is a plate C that shares a divergent boundary with both plates A and B. So let us start by taking points on B at time *t* and rotating them back to their position relative to plate C at time 0 (today); this is a variation on equation (3):

$$\vec{X}_C(0) = \mathbf{R}_{CB}^{t\to 0}\vec{X}(t) \tag{7}$$

Now we know where our points were with respect to plate C, but plate C has moved from plate A:

$$\vec{X}_A(0) = \mathbf{R}_{AC}^{t\to 0}\vec{X}(t) \tag{8}$$

The *X*(*t*) on the right here represents points with respect to plate C, so we can combine (7) and (8) to get

$$\vec{X}_A(0) = \mathbf{R}_{AC}^{t\to 0}\mathbf{R}_{CB}^{t\to 0}\vec{X}(t) \tag{9}$$

A comparison of equations (9) and (3) shows that the product of the two rotation matrices is equivalent to the single rotation tensor connecting A and B. We thus can recover the net motion between plates A and B. Note that this is not commutative: you have to combine rotations in the proper order.

From this information, we can go a step farther and now derive the net Euler pole for the motion of plate B relative to plate A. If we reexamine the components of the rotation matrix **R** in (2), we see that the components of the Euler unit vector can be trivially determined (if we know the net rotation) by adding or subtracting elements of the net rotation matrix. So, for instance,

$$R_{xy} - R_{yx} = -2z\sin\omega \quad (10)$$

where x, y, and z are the components of the Euler pole. To get the magnitude of the rotation, ω, one possibility is to use the second invariant of the matrix:

$$R_{xx}R_{yy} + R_{zz}R_{yy} + R_{zz}R_{xx} - R_{xy}R_{yx} - R_{zy}R_{yz} - R_{zx}R_{xz} = 2\cos\omega + 1 \quad (11)$$

(The subscripts will correspond to the lead term in (2)).

With these two precepts in mind, you can develop a plate circuit connecting distant plates. Of interest to us is the motion of the plates in the Pacific Ocean, including the modern Pacific but also the ancient Farallon, Kula, Vancouver, and Resurrection plates. Again, there are two issues of concern: one is that only tiny and very young fragments exist of these ancient plates (so we lack the magnetic anomalies on those plates to match up with equivalent anomalies on the Pacific plate), and the other is that you have to cross a bunch of spreading centers to get from North America to the plates in the Pacific basin, and some of these spreading centers have uncertain histories.

First, the issue with the missing plates. In general, we observe that magnetic anomalies are symmetrically distributed across a spreading center: the rate of creation of plates on either side of the spreading center is generally the same. The exceptions usually occur in places where the ridge jumps into one plate or the other, and the patterns of such behavior usually can be observed in the magnetic anomalies. If we assume symmetry, then the rotation that moves the anomaly on the Pacific plate produced at a ridge between Pacific and, say, Farallon from time 1 to that produced at time 2 is half the total rotation of Farallon relative to the Pacific. This is actually a stage pole (that is, the net rotation over an ancient time interval) and so is used a bit differently in practice than the normal observations of the rotation between anomalies of the same age on different plates (it is equivalent to our $\mathbf{R}_{AB}^{1\to2}$ rotations above).

It is extremely important to recognize two things here: first, that we have to assume symmetric spreading, and the second is to presume there isn't another plate between Farallon (or Kula or Resurrection…) and North America. If we had an additional subduction zone with some backarc spreading, we might know nothing of what was happening at the margin of North America. During the Late Cretaceous, the spreading ridge on the east side of the Pacific plate was near the middle of the Pacific Ocean basin, allowing plenty of room for additional boundaries to the east.

The second issue has been more troublesome. The plate circuit connecting North America to the Pacific runs across to Africa, then through Antarctica and on to the Pacific. The main issue is that in the early Tertiary and latest Cretaceous, there are uncertainties in the motions across Antarctica (there was a plate boundary between East and West Antarctica of uncertain motion) and there are ambiguities in motions within the Pacific plate. Although some recent work (Doubrovine and Tarduno papers in JGR, 2008) has refined these issues and added a separate circuit from Africa through Australia to the Pacific, an alternative that has often been considered attractive was to connect plates through hot spots.

Hot spots remain controversial both in terms of their origin (plumes originating at the core-mantle boundary, or somewhere higher, or just migratory melt anomalies) and their relative stability. Observationally, these are volcanic centers that migrate in one diretion over time. Jason Morgan most notably advocated that a large number of migratory volcanic centers were the signatures of plumes originating deep in the mantle and effectively fixed to the deep Earth and unaffected by plate motions. As such, the assumption made was that plumes in the Pacific did not move with respect to plumes under Africa. Thus if you pretend that plumes form a "plate" (a reference frame with no internal deformation), you could connect North America through Africa and its plumes to the Pacific and its plumes and so avoid worries in Antarctica and the south Pacific altogether. This was the approach taken by David Engebretson in the mid-1980s under the guidance of Allan Cox at Stanford. The resulting plate model has dominated discussion of late Mesozoic to early Tertiary plate environments in western North America.

Unfortunately, there was never any theoretical support for the supposition that plumes do not move relative to one another. Numerical and physical simulations of plumes in a convecting medium show that they should move relative to one another, probably at rates of tens of percent of relative plate motions, if not occasionally higher. If plates reorganize, such that the motions of large plates relative to one another shift to being about very different poles, one expects major shifts in the positions of any plumes under one plate relative to those under others. Observationally, while there is a lot of consistency between hotspot tracks within a single plate, there is not consistency over multiple plates and there are inconsistencies between tracks across large plates like the Pacific. These errors are generally additive and make the use of hotspot-based reconstructions shaky at best in the distant past. Finally, actually getting ages of the positions of hotspots is difficult in practice as volcanic edifices have finite lifetimes. Because of all of these difficulties, plate reconstructions relying on fixed hotspots should be treated as quite uncertain; unfortunately, the literature is full of papers accepting even small changes in plate motion from the Engebretson models as significant and worthy of interpretation.