Explanation of the coordinate systems used to plot raypaths

July 17, 2009

1 Introduction

There are three different plot projections we use for plotting raypaths and for contour plotting. In the case of raypath plots, these will be projections on a vertical or horizontal plane. In the case of contour plots, these will be vertical or horizontal slices. The three projections (or slices) are:

- A projection on the ground (or horizontal slice) (W81=2)
- A projection on a vertical plane (or vertical slice) as a curvilinear plot to account for the Earth's curvature (W81=3)
- A projection on a vertical plane (or vertical slice) (W81=4) as a rectangular plot with height as a function of horizontal position.

In all three cases, the user can specify the plotting region in a nearly arbitrary way. In all three cases, the user chooses the latitude and longitude of the left edge of the plot and also of the right edge of the plot.

For the projections on a vertical plane (or for a vertical slice for a contour plot), the user also chooses the height of the bottom of the plotting region and the height of the top of the plotting region. For contour plots in a horizontal slice, the user chooses the height of the slice.

2 Horizontal projections (W81=2)

For the projections on the ground, imagine a great circle drawn on the surface of Earth that connects the longitude and latitude of the left edge of the plot with the longitude and latitude of the right edge of the plot. Any point on a raypath is first projected straight down onto the surface of the Earth. That point on the surface of the Earth is then projected onto the great circle mentioned above by moving it normal to that great circle. We then plot the distance from that great circle versus the position on the great circle to where that point was projected. In addition, the user can specify an expansion factor (W82) to which to multiply the distance from the great circle.

For contour plots in a horizontal slice, the method is essentially the same.

3 Projections on a vertical plane, curvilinear plot (W81=3)

For explaining the curvilinear projections on vertical slice, it is best to refer to the following three figures. Figure 1 shows the vertical slice for which we wish to project a raypath. This vertical slice follows a great circle that connects the longitude and latitude of the left edge of the plot to the longitude and latitude at the right edge of the plot. The corresponding central-Earth angle from the left edge of the plot to the right edge of the plot is 2α, as shown in figure 1. The radius of the Earth is R_E, which is shown, along with an arc that represents the surface of the earth. The plotting region is also shown. The height of the bottom of the plotting region is h_B, as shown. The height of the top of the plotting region is h_T, as is also shown.

The main advantage of a curvilinear plot of raypaths is that it shows an accurate representation of the raypaths. However, because for propagation over long ranges, the vertical extent of raypaths is often much less than the horizontal extent, it is often desirable to expand the vertical scale by some factor (W82) chosen by the user. Choosing W82 to be one gives no expansion (that is, it gives an accurate representation of the raypaths). Although we usually want to choose W82 to be larger than one, the program allows values less than one also.

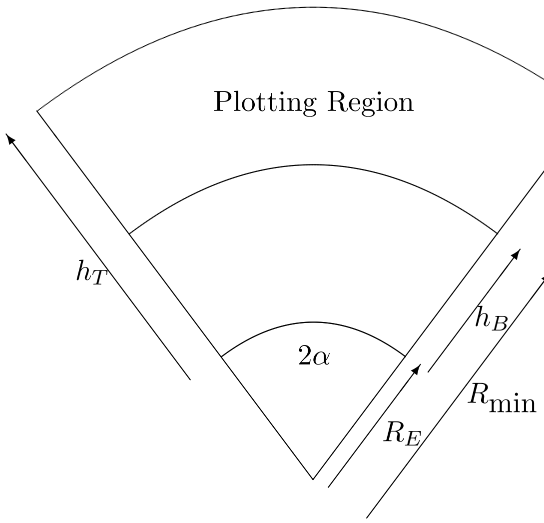

Figure 1: One of the coordinate systems used for plotting on a vertical plane, showing the vertical plane in correct proportion. The radius of the Earth is R_E, which is shown, along with an arc that represents the surface of the earth. The plotting region is also shown. The height of the bottom of the plotting region is h_B, as shown. The height of the top of the plotting region is h_T, as is also shown. The central-Earth angle from the left of the plot to the right of the plot is 2α.

In expanding the vertical scale, we hold the bottom of the plotting region fixed. That is, the radial distance of the bottom of the plotting region R_{min} will remain constant for the expansion. The effect of such an expansion is shown in figure 2 for an expansion factor $P = W82 = 1.5$. As can be seen, the height of plotting region has expanded to equal $P \times (h_T - h_B)$ for $P = W82 = 1.5$. As can be seen in figure 2, there is a radial expansion away from the bottom of the plotting region in both directions (up and down), but the effect of the expansion downward will never be seen in the plots because that is outside of the plotting region.

One point on a raypath is shown in the plotting region in figure 2 having a horizontal position x from the center of the plotting region. This is an angle in radians. The vertical position of that point is y, which is a distance in kilometers from the center of curvature of the curvilinear plot to the raypoint. If the expansion factor were equal to one, it would be a distance from the center of the earth.

Although a raypath plot or a contour plot will be seen as shown in the plotting region of figure 2, in programming the plotting, a plot is treated as though we were making a rectangular plot as shown in figure 3. We do this because the contour program makes contours on a rectangular grid, and for this to work most efficiently for the curved plotting region, we want to make that curved plotting region conform to a rectangular region when making the contours.

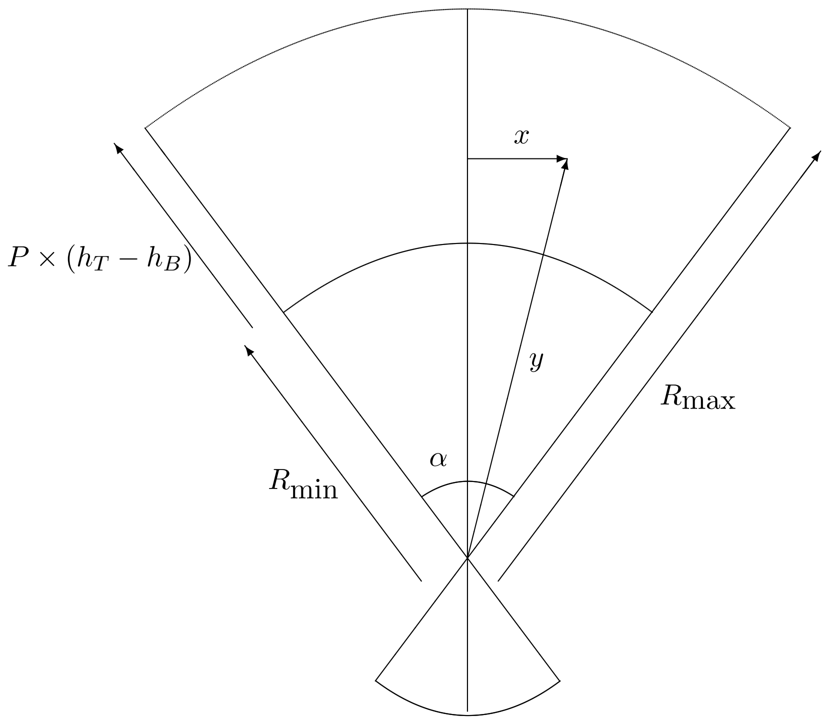

Figure 2: One of the coordinate systems used for plotting in a vertical plane, showing the effect of stretching the plotting region by a factor P in the radial direction. We have used $P = 1.5$ in this example. This is how the plotting region would appear in the plot. One point is shown in the plotting region, along with x and y values labeled. α is the central-Earth angle from the left or right edge of the plot to the center of the plot. The calculation of $R_{\max}$ is after statement 100 in SUBROUTINE RAYPLT in file s3.for. The calculation of y including the expansion factor is after statement 150 in SUBROUTINE RAYPLT. The calculation of the coordinates to plot is before statement 49 in SUBROUTINE PLOT in file s6.for.

Figure 3 shows that rectangular representation. The horizontal scale is central-Earth-angle in radians. The vertical scale is in kilometers.

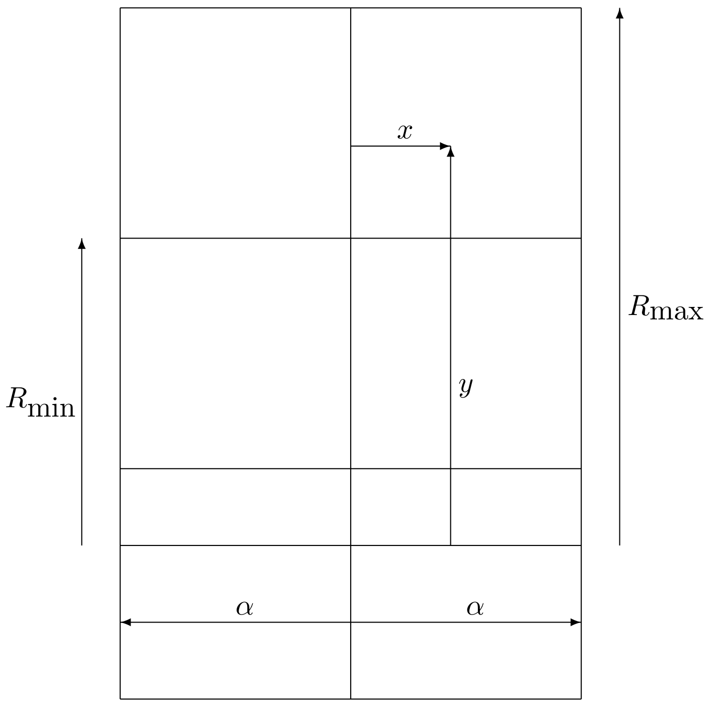

Figure 3: One of the coordinate systems used for plotting in a vertical plane, in which the radial coordinate and the central-Earth angle are shown in a rectangular plot. One point is shown in the plotting region, along with x and y values labeled.

4 Projection on a vertical plane as a rectangular plot (W81=4)

For the rectangular plot projection on a vertical plane, the plotting region is simply plotted as a rectangular plot similar to that shown in figure 3, except that the expansion factor is ignored. The program simply chooses a convenient scale.

5 Additional notes

When it is necessary to alter some of the labeling of the plots, it is first necessary to determine which routines to alter, and find out where those routines are. To help in doing that, we list here some of that information. This list is correct at least for projections on a vertical plane.

The listing of the models used and the date is done in SUBROUTINE LABPLT. The axis captions and the printing of the longitude and latitude of the left edge and right edge of the plot is done in SUBROUTINE PLTANOT. The tick mark labels are done in SUBROUTINE PLTLB. All three of these subroutines are in raytracing/g/s6.for.