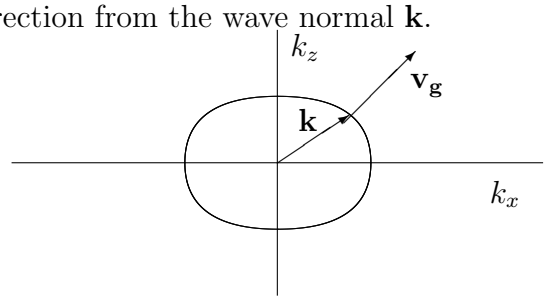
Tutorial on ray tracing dispersion relations

A dispersion relation gives the relationship between the frequency $\omega$ and the three components of the wave vector $\mathbf{k}$. A dispersion relation is the single most important formula to characterize a wave in that it allows most of the important properties of a wave to be calculated, such as phase velocity (the velocity of a wave front), group velocity (the velocity of a wave packet), and refraction (bending of the wave).

It is often instructive to show dispersion relation curves for a fixed frequency. The following two examples show such curves, showing the relationship between the horizontal and vertical components of the wave vector $\mathbf{k}$ for a fixed frequency. Varying the frequency would give a family of such curves.

The first graph shows a dispersion relation curve for an acoustic wave. $k_x$ is the horizontal component of the wave vector, and $k_z$ is the vertical component. The vector $\mathbf{k}$ (which goes from the origin to the dispersion relation curve) gives the direction of the normal to a wave front. The vector $\mathbf{v_g}$ gives the direction of the group velocity, which is normal to the dispersion relation curve. Notice how the group velocity $\mathbf{v_g}$ is in a slightly different direction from the wave normal $\mathbf{k}$.




The next graph shows a dispersion relation curve for a gravity wave. Notice how the vertical component of the group velocity is in the opposite direction from the vertical component of the wave-normal direction.